Percolation – substructures

Backbone, dead ends, red bond and blobs

- The fractal dimensions d_f and $d_{\min}$ are not enough to characterize percolation clusters

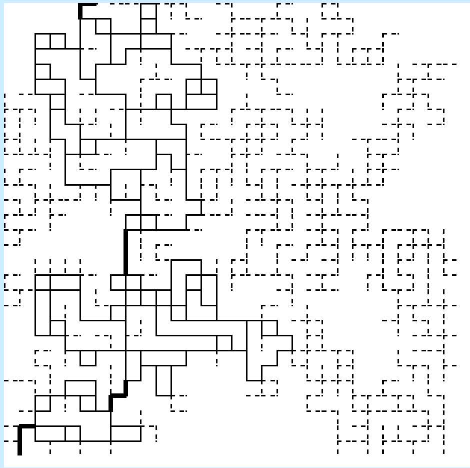
- We impose a voltage drop between two sites on the infinite cluster

- The backbone includes all bonds which carry current

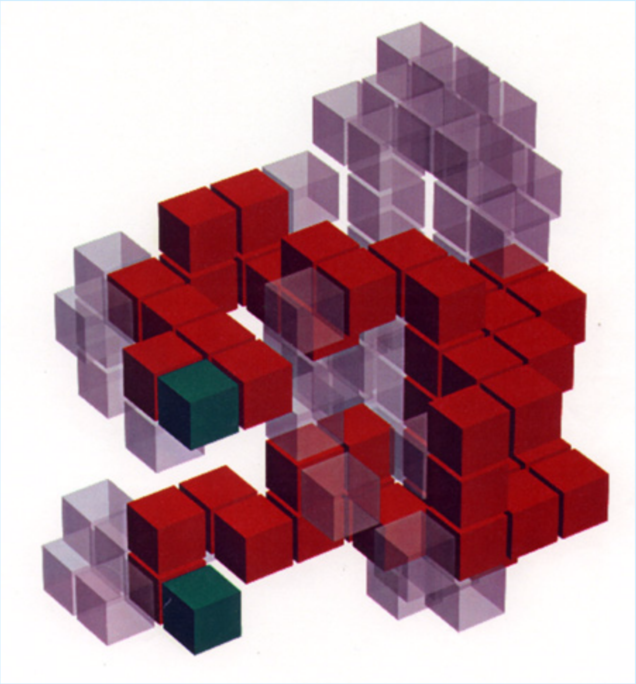
- The dead ends are the parts that do not carry any current

- The red bonds (called also singly connected bonds) are those links that carry all current (dark black). When they are cut the current stops

- Blobs are the parts of the backbone left after removing the red bonds



Backbone



- The backbone for site percolation in d=3 is shown here
- Green sites have a voltage drop
  The backbone is shown in red
  The dead ends are shown in gray
- The backbone in any dimension is only a small (zero) fraction of the infinite cluster. Its fractal dimension is smaller:

$$M_{BB} \sim R^{d_B}, \; d_B < d_f$$ (see Table)

- Thus, most of the mass of the cluster is in the dead ends.

Backbone

- The values of the fractal dimension of the backbone, d_B are known only numerically (for $2 \le d \le 5$). Analytical derivation exists only for $d = d_c = 6$ (see Table)

- $d_{\min}$ on the backbone is the same as $d_{\min}$ on the percolation cluster. This is since from every two sites one can generate a backbone and the shortest path will be on it.

- d_l^B - the chemical dimension of backbone is not the same as d_l for the cluster.

$$d_l^B = d_B / d_{\min} \quad \text{(while} \quad d_l = d_f / d_{\min})$$

d	2	3	6
d_f	91/48	2.53	4
$d_{\min}$	1.1307	1.374	2
d_{red}	3/4	1.143	2
d_h	7/4	2.548	4
d_B	1.6432	1.87	2

Red bonds

- The fractal dimension of red bonds is known analytically (Coniglio 1982)

- For any d: the number of red bonds is

$$n_{red} \sim (p - p_c)^{-1}$$

- Since the correlation length $x \sim (p - p_c)^{-n}$ it follows

$$n_{red} \sim x^{1/n}$$

- When looking at $r < x : n_{red} \sim r^{1/n}$

- Thus, the fractal dimension of red bonds is for all *d*

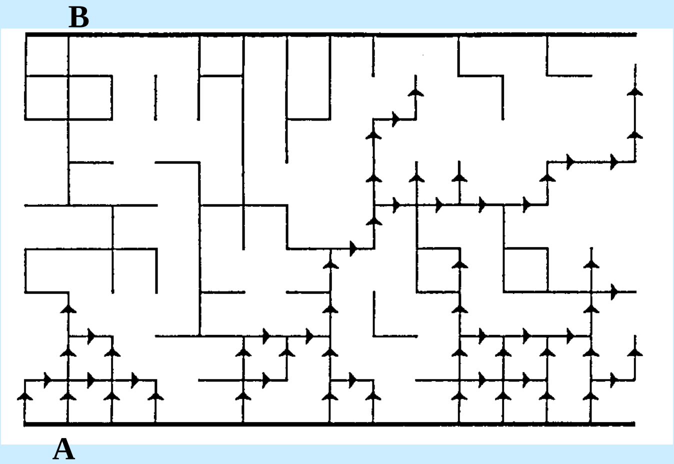
$$d_{red} = \frac{1}{n}$$

Directed Percolation

→Bond percolation on a square lattice

→Each bond has a direction towards
x>0 or y>0

→Current can flow only in the arrow
direction

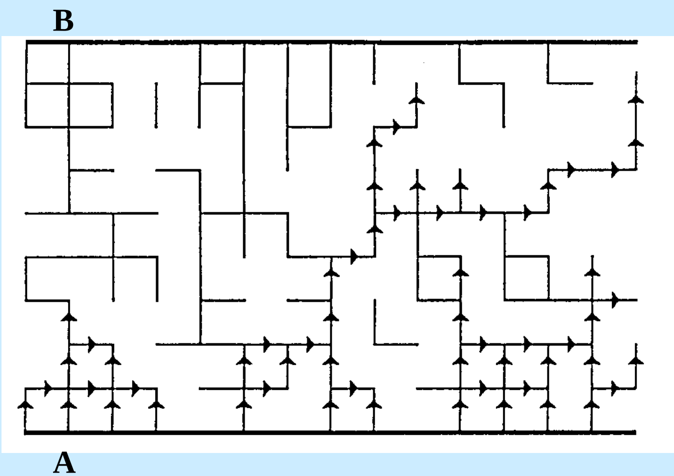


❖ Model for forest fire spreading under the influence of a wind

❖ Model for current in random diodes network

❖ Model for surface growth

Directed Percolation



➢ There is a critical $p=p_c$ of directed bonds

➢ For $p<p_c$ no current flow from A to B

For $p>p_c$ current can flow!

➢ p_c is larger than p_c of isotropic percolation

For square lattice p_c =0.6447 (instead of p_c =0.5)

For triangular lattice p_c =0.479 (instead of p_c =0.35)

➢ The reason is that one needs to create a path without overhangs, d_{min}=1

(compared to d_{min}=1.13 in regular percolation)

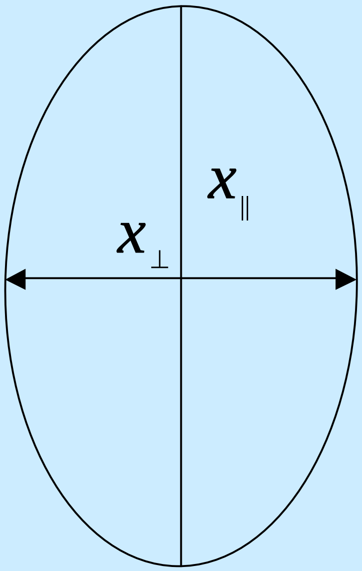
Directed Percolation- Two correlation lengths

- The structure of directed percolation clusters is anisotropic
- Two correlation length:

$x_{\parallel}$ -in the percolation direction (x>0, y>0)

$x_{\perp}$ -perpendicular to percolation direction

$$x_{\parallel} \sim \left|P - P_c\right|^{-n_{\parallel}} \qquad x_{\perp} \sim \left|P - P_c\right|^{-n_{\perp}} \qquad n_{\perp} < n_{\parallel}$$

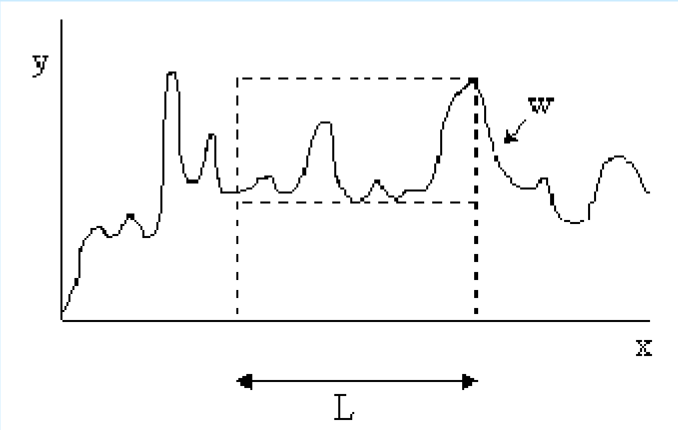


- The clusters are therefore self affined
- For $d = 2$: $\quad n_{\perp} \cong 1.097 \qquad n_{\parallel} \cong 1.733$
- A directed path will have a width $w \propto L^{a}$

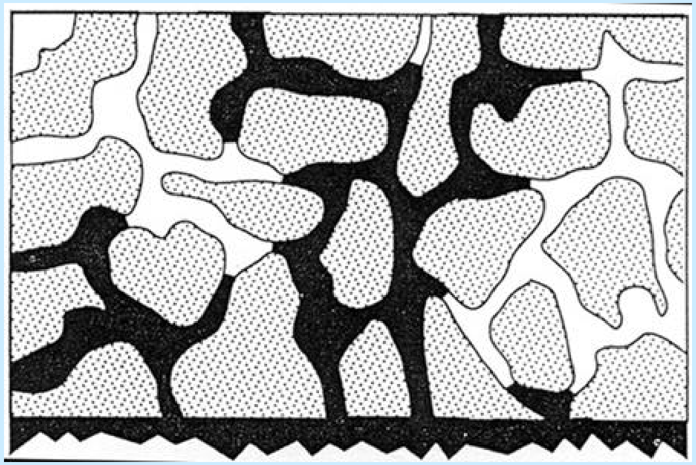
$$w \sim x_{\perp} \qquad L \sim x_{\parallel}$$

$$x_{\perp} \propto \left|P - P_c\right|^{-n_{\perp}} \propto x_{\parallel}^{\,n_{\perp}/n_{\parallel}} \sim x_{\parallel}^{\,0.63}$$

Thus $\underline{w \sim L^{0.63}}$



Invasion Percolation



- ❖ Flow of water into a porous media full of oil
- ❖ To extract oil from oil field usually one inserts water with high pressure in one hole and oil comes out from another hole
- ❖ Water and oil are incompressible fluids therefore when water invades into the rock oil comes out.

Invasion Percolation Model

❖ A lattice LxL full of oil

❖ Water invades from left bar

❖ Random numbers represent the resistance to invasion

❖ Water invades step by step in the smallest resistance sites

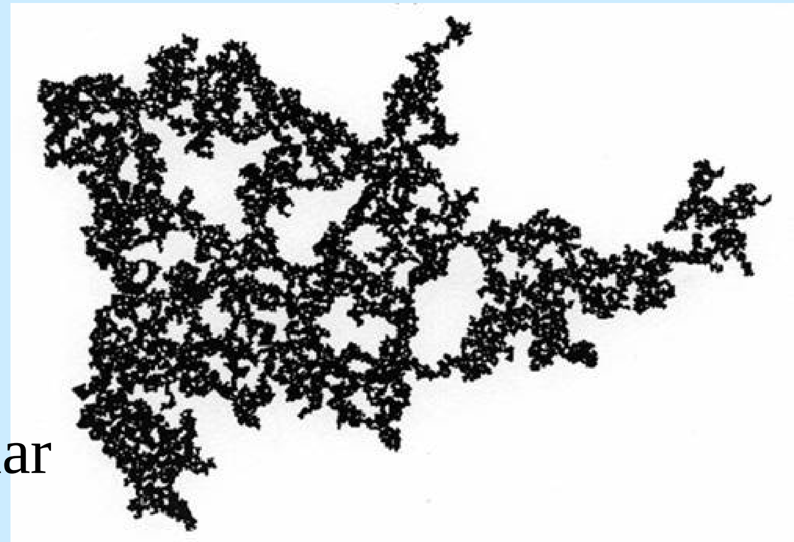
❖ This model is equivalent to PRIM and KRUSKAL algorithms for finding the "minimum spanning tree"

0.55	0.01	0.64	0.16	0.88
0.33	0.81	0.84	0.19	0.23
0.38	0.25	0.09	0.42	0.65
0.91	0.19	0.50	0.22	0.40
0.09	0.02	0.47	0.28	0.30

0.55	0.01	0.64	0.16	0.88
7	0.81	0.84	9	0.23
6	4	5	8	0.65
0.91	3	0.50	0.22	0.40
1	2	0.47	0.28	0.30

Invasion Percolation

❖ Since oil and water are incompressible liquids, regimes of oil surrounded by water can not be invaded any more



❖ Oil can be trapped in the porous media

❖ For d=2 d_f=1.82 < d_f=1.896 of regular percolation

d_{min}=1.22 > d_{min}=1.13 of regular percolation

❖ These changes are due to the trapping

❖ For d=3 d_f =2.5 close to regular percolation