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Effective medium approximation for hopping conductivity and Josephson junctions

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Abstract

In the framework of self-consistency effective medium approximation, we derive an expression for the effective resistivity of the Miller–Abrahams resistor networks with hopping conductivities, and we compare it with results of numerical simulations of such networks. In our approach we modify the effective medium approximation to include the presence of the Josephson junctions in granular (low- and high- T_c) superconductors.

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1. Introduction

Nanoscopic granular materials cannot be treated as classical composites [1], since the typical size of their grains (of the order ~15–20 nm) requires the quantum-mechanical description. Despite the big progress made in the theory of disordered and percolating media, the main tools for studying such systems remain the effective medium approximation (EMA) and the resistor network numerical simulations (RNNS). In this work we modify both EMA and RNNS in order to take into account the quantum effects including quasi-particles tunneling and the presence of Josephson junctions. It enables us, e.g., to calculate the temperature dependences of the superconductor–insulator transition in granular superconductors.

2. Quasi-particle tunneling

The most common and convenient expression for the inter-granular hopping resistance R_{ij} (between the ith and jth grains) is the well-known formula [2]

$$R_{ij} = R_0 \exp(r_{ij}/r_0 + \varepsilon_{ij}/k_B T), \quad (1)$$

where r_{ij} is the distance between two grains, r_0 is the scale over which the wave-function decays outside the grain, $\varepsilon_{ij} = (|E_i| + |E_j| + |E_i - E_j|)/2$ is the activation energy and R_0 is a scaling resistance. In order to perform numerical simulations, we express the random distance between grains as $r_{ij} = 2\bar{l} \cdot \xi_{ij}$, where ξ_{ij} is a random number taken from a uniform distribution in the range (0,1), and $\bar{l}$ is the mean distance between metallic grains [3]. At sufficiently high temperatures T (when the thermal hopping term $\varepsilon_{ij}/k_B T$ is small compared with the spatial one r_{ij}/r_0), the expression for R_{ij} can now be rewritten as [3]

$$R_{ij} = R_0 \exp(\kappa \cdot \xi_{ij}), \quad (2)$$

where $\kappa \equiv 2\bar{l}/r_0$ can be interpreted as the dimensionless mean tunneling distance or as the degree of disorder (the lower the density of the deposited grains, the larger is κ). The latter interpretation is since for larger κ, for two values of ξ_{ij} the values of R_{ij} are more separated [3].

Further treating of the system can be performed by the construction of a Miller–Abrahams resistor network [2]. In Ref. [4] an analytical expression (derived in the percolation approach) is presented for the average effective resistance R_e of such resistor networks. In the two-dimensional (2D) case, it takes the form

$$R_e \simeq R_0 e^{p_c \kappa}, \quad (3)$$

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where p_c is the percolation threshold. Ref. [5] shows that in 2D, in the limit $\kappa \to \infty$, Eq. (3) is exact.

It is easy to show [3] that in the case of the 2D random resistor bond network (for which $z = 4$ and $p_c = 0.5$), Eq. (3) follows immediately from the Keller–Dykhne theorem [6], and is exact for an arbitrary κ. According to this theorem, the effective conductivity σ_e of n-component composite (with local conductivities σ_i) and the analogous quantity of the dual system (with $\sigma_i' = \sigma_i^{-1}$) are connected as follows: $\sigma_e(\sigma_1, \sigma_2, \ldots, \sigma_n)\sigma_e(\sigma_1^{-1}, \sigma_2^{-1}, \ldots, \sigma_n^{-1}) = 1$. Rewriting Eq. (2) as $\sigma_{ij} \sim e^{-0.5\kappa} e^{-\kappa[\xi_{ij}-0.5]} = e^{-0.5\kappa} e^{-\kappa\tilde{\xi}}$ (where $\tilde{\xi} \in (-0.5, 0.5)$) and taking into account that σ_e is a homogeneous function (see e.g., Ref. [1]), i.e., that any factor can be taken out from the brackets, we get $e^{-0.5\kappa}\sigma_e([e^{-\kappa\tilde{\xi}}])e^{0.5\kappa}\sigma_e([e^{\kappa\tilde{\xi}}]) = 1$. Here $[\ldots]$ means that the values $e^{\kappa\tilde{\xi}}$ are arranged in the consequently increasing order. Since sequences $[e^{-\kappa\tilde{\xi}}]$ and $[e^{\kappa\tilde{\xi}}]$ are self-dual, $\sigma_e([e^{-\kappa\tilde{\xi}}]) = 1$. Multiplying this by $e^{-0.5\kappa}$, we get Eq. (3).

The effective resistance R_e can be derived also in an EMA framework [7]. If the local resistivities, R, are distributed continuously in a range $R_{min} \leqslant R \leqslant R_{max}$ according to some distribution function $f(R)$, then R_e can be found by taking the integral

$$\int_{R_{min}}^{R_{max}} f(R)\left(\frac{R - R_e}{aR + R_e}\right) dR = 0, \tag{4}$$

where $a = z/2 - 1$ and z is the number of bonds (neighbors) for each node of the network. If ξ_{ij} in Eq. (2) is uniformly distributed between 0 and 1, then $f(R) = 1/\kappa R$ (see e.g., Ref. [4]). Taking the integral (4), we obtain [8]

$$R_e = R_0 e^{\kappa p_c}\left(\frac{1 - p_c}{p_c}\right)\left[\frac{1 - e^{-\kappa p_c}}{1 - e^{-\kappa(1-p_c)}}\right], \tag{5}$$

where $p_c = 1/(1 + a) = 2/z$. In the limit $p_c \to 1$, Eq. (5) reduces to $R_e = R_0(e^{\kappa} - 1)/\kappa$, and the latter tends to R_0 when $\kappa \to 0$. For a square lattice ($z = 4$), Eq. (5) coincides with Eq. (3).

To verify Eq. (5), we built bond-percolating Miller–Abrahams like resistor networks [2,3], assuming the conductivity of each resistor is given by Eq. (2). We wrote the Kirchhoff equations for each site, solve the system of the obtained coupled linear equations for the voltages at every site and calculate the total effective resistance R_e for 2D networks [3]. In Fig. 1, we show a semi-log plot of the dependences R_e vs. κ, obtained from both numerical simulations and analytical expressions (5).

3. Resistance distribution

As follows from Eqs. (3), (5), the effective resistance of the system with wide range distribution (1) does not depend on the size of the system, L. However, the other properties of this system can depend on L. For example, we found [3] that the distribution function $P(R)$ (i.e., the probability that the total resistance of the system is R) can

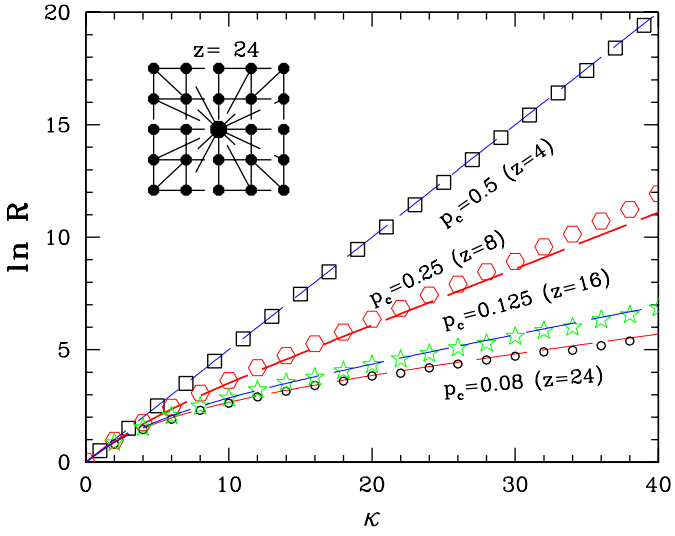

Fig. 1. A semi-log plot of the averaged resistance R_e vs. κ for samples with different nearest-neighbors z. The results of numerical simulations (shown by empty symbols) are in agreement with the analytical expression (5) obtained with EMA (shown by lines). Presented are the cases with $z = 24$ ($p_c = 0.08$), $z = 16$ ($p_c = 0.125$), $z = 8$ ($p_c = 0.25$), and $z = 4$ ($p_c = 0.5$), respectively. Inset shows the lattice with $z = 24$ neighbors. The central grain in each unit cell (shown by larger filled circles) is connected with z neighbors (shown by smaller circles).

be approximated by the log-normal form [3]

$$P(R) \simeq \frac{1}{\sqrt{2\pi}\mu R}\exp\left[-\frac{\ln^2(R/\bar{R})}{2\mu^2}\right], \tag{6}$$

where $\mu = \alpha(p_c\kappa)^{\nu}/L$ for both 2D and 3D cases (in 2D is found that $\alpha \simeq 0.5$). Here ν is the critical exponent in the percolation correlation length $\xi \propto (p - p_c)^{-\nu}$ (in 2D $\nu = \frac{4}{3}$, while in 3D $\nu \simeq 0.88$ [4,9]). In fact, Eq. (6) includes also the weak disorder case ($\mu < 1$), (in this case Eq. (6) reduces to the Gaussian form), while at extreme disorder ($\mu \gg 1$) the exponent function in Eq. (6) tends to 1, and $P(R)$ transforms to the power-like dependence $\sim 1/R$. The value μ can be regarded as a criterion for weak and strong disorder.

From Eq. (6) it follows that at $\mu \to 0$, the distribution function $P(R)$ reduces to a delta-function

$$\lim_{\mu\to 0} P(R) = \frac{1}{R}\lim_{\mu\to 0}\frac{1}{\sqrt{2\pi}\mu} e^{-\ln(R/R_e)^2/2\mu^2}$$
$$= \frac{1}{R}\delta(\ln R - \ln R_e) = \delta(R - R_e).$$

Therefore, at $\mu \to 0$ (i.e., $\kappa \to 0$ or $L \to \infty$) the total resistivity of the system is exactly $R_0 e^{p_c\kappa}$ and has no size dependence: $\lim_{\mu\to 0} \int RP(R)\, dR = R_e$.

4. Superconductor–insulator transition

In this section we present a model for studying the superconductor–insulator transition [10,12] using EMA for both low- and high-T_c granular superconductors.

4.1. Low-T_c superconductors

The resistance between two grains is governed either by the Josephson junction coupling or by the quasi-particle tunneling. The latter is described by Eq. (1), where ε_{ij} is related to the superconducting gap [11], $\Delta_{ij}(T)$, while the Josephson junction coupling is determined by the Boltzmann thermal energy, $k_B T$, and the Josephson binding energy, E_J, which is given by $E_J = (\hbar/2e)J_J = \alpha[\Delta(T)/R_{ij}^{(n)}]\tanh[\Delta(T)/2k_B T]$, where J_J is the Josephson current, $\alpha = \pi\hbar/4e^2 \cong 3.25\,\mathrm{k\Omega}$ and $R_{ij}^{(n)}$ is the normal resistance between the grains [11,13]. The condition for two neighboring grains to be Josephson coupled is $zE_J \geqslant k_B T$, where z is the number of the nearest-neighboring grains [11]. Hence, we can determine the resistance criterion for the Josephson coupling [13]:

$$R_{ij}^{(n)} \leqslant R_J \equiv z\alpha\delta(T)\tanh[\delta(T)/2], \tag{7}$$

where $\delta(T) = \Delta(T)/k_B T$. We can rewrite EMA equation (4) in the form which takes into account the Josephson coupling

$$\int_{R_{\min}}^{R_{\max}} \theta(R\mathrm{e}^{-\Delta/k_B T} - R_J) f(R)\left(\frac{R - R_e}{aR + R_e}\right) \mathrm{d}R - \int_{R_{\min}}^{R_{\max}} \theta(R_J - R\mathrm{e}^{-\Delta/k_B T}) f(R)\,\mathrm{d}R = 0, \tag{8}$$

where R_J is determined by Eq. (7). In the first integral we calculate the cases when R is larger than necessary for the Josephson coupling $R^{(n)} > R_J$ (i.e., when $\theta(R^{(n)} - R_J) = 1$, where θ is the Heaviside function). Note, that $R^{(n)} = R_0 \exp(\kappa \cdot \xi) = R\exp[-\Delta(T)/k_B T]$, see Eqs. (1)–(2). In the second integral we consider the opposite situation, i.e., when $\theta(R_J - R^{(n)}) = 1$. In this case Josephson coupling exists and R in the brackets should be taken as zero ($R \to 0$). If ξ in Eq. (2) is uniformly distributed between 0 and 1, then $f(R) = 1/\kappa R$, and R is varied in the range $R_0\mathrm{e}^{\delta} \leqslant R \leqslant R_0\mathrm{e}^{\delta+\kappa}$. From Eq. (8) we get [14]

$$R_e = \frac{1 - p_c}{p_c}\frac{(R_0\mathrm{e}^{\tilde{\kappa}p_c} - R_J)\mathrm{e}^{\Delta/k_B T}}{1 - \mathrm{e}^{-\tilde{\kappa}(1-p_c)}}, \tag{9}$$

where R_J is given by Eq. (7).

4.2. High-T_c superconductors

In contrast to the low-T_c superconductors described above, the high-T_c granular ceramics have wide dispersion of the intrinsic transition temperature T_c among different grains [12]. This allows us to approximate the granular sample as a mixture of normal and superconducting particles and to use two-component EMA [7,12]. Our approach takes into account both the strong geometrical anisotropy of the grains (with aspect ratio ~100) as well as the presence of the Josephson junctions. Thus, instead of the composite with conductivity tensor $\hat{\sigma}^{(2)}$ and oblate shape of the inclusions, we can consider a composite with spherical inclusions but a different conductivity tensor $\hat{\mu}^{(2)}$, satisfying [15]

$$\mu_{\alpha\alpha}^{(2)}/\mu_{\beta\beta}^{(2)} = (L_\beta/L_\alpha)^2(\sigma_{\alpha\alpha}^{(2)}/\sigma_{\beta\beta}^{(2)}), \tag{10}$$

where L_α is a characteristic size in the direction $\alpha = z, y, z$ (e.g., the semi-axes of the ellipsoids a, b, c) in the initial system. We then construct a random 3D resistor network in order to perform Monte Carlo simulations of the conductor/superconductor composite with host conductivity tensor $\hat{\mu}_2$ and calculate the effective conductivity $\hat{\mu}_e$ of the considered system. The effective conductivity tensor $\hat{\sigma}_e$ can then be found as [15]

$$\sigma_{\alpha\alpha}^{(e)} = (\sigma_{\alpha\alpha}^{(2)}/\mu_{\alpha\alpha}^{(2)})\mu_{\alpha\alpha}^{(e)}. \tag{11}$$

The knowledge of the local currents on each inter-grain junction enables us to compare them with the critical values of the Josephson current J_{Jc}, and simulate the current- and the magnetic field-dependences of the resistance transition of the entire sample [16].

5. Conclusion

In summary, we modify the EMA for the case of hopping conductivity with many neighbors and the presence of Josephson junctions. Our analytical expressions are in agreement with the results of our numerical simulations as well as with experimental measurements [10,12,14].

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