Long-term memory in earthquakes and the distribution of interoccurrence times

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Abstract – We study seismic records in regimes of stationary seismic activity in Northern and Southern California. Our analysis suggests that the earthquakes are long-term power law correlated with a correlation exponent \(\gamma\) close to 0.4. We show explicitly that the long-term correlations can explain both the fluctuations of magnitudes and interoccurrence times (between events above a certain magnitude \(M\)) and, without any fit parameter, the scaling form of the distribution function of the interoccurrence times in the seismic records, recently obtained by Corral (Phys. Rev. Lett., 92 (2004) 108501).

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Introduction. – Understanding the mechanism behind the complex spatio-temporal behavior of earthquakes is one of the major challenges in science (for reviews, see [1–4]). In phenomenological models like the ETAS model [5–7], it is assumed that there is no temporal correlation between the magnitudes of subsequent earthquakes. It is an intriguing question with great relevance for the predictability of earthquakes if this assumption is justified or if temporal correlations between the magnitudes exist and have to be taken into account [8–11]. In particular, since long-term memory is quite common in nature and occurs, for example, in climate [12–16], physiology [17,18], and financial markets [19], it is of great interest to find out if it exists also in seismic activity.

Recently, considering various tectonic environments as well as mainshocks and aftershocks as part of essentially one unique process, Bak et al. [20] and Corral [21–23] analyzed the interoccurrence times between earthquakes with amplitudes greater than \(M\) in a large number of spatio-temporal areas of varying sizes. While Bak et al. [20] concentrated on the distribution of the interoccurrence times in California and obtained a unified scaling law for the spatio-temporal set of data (see also [24,25]), Corral [21–23] studied the interoccurrence times in a large number of spatial areas of various sizes with stationary seismic activity. He found that independently of the considered area and independently of the threshold \(M\), the probability distribution function (pdf) of interoccurrence times scales with the mean interoccurrence time \(\tau_M\) as

\[ D_M(\tau) = \frac{1}{\tau_M} f(\tau/\tau_M), \]  

where \(f(v)\) is a universal scaling function which does not depend on \(M\) and (apart from very small \(v\) values) can be well approximated by the Gamma distribution,

\[ f(v) = C v^{-(1-\gamma)} \exp(-v^\delta/B), \]  

with \(C \approx 0.5\), \(B \approx 1.58\), \(\delta \approx 0.98\), and \(\gamma \approx 0.67\) [22], see also [26]. It is interesting that according to Shcherbakov et al. [27] this kind of distribution also holds for pure aftershock sequences, but with different parameters and exponents. In a recent analytical study of the ETAS model, Saichev and Sornette [7] obtained a different form for \(f(v)\),

\[ f(v) = (a_n \rho \theta v^{-1-\theta} + (1 - n + na \rho \theta v^{-\theta})^2 ) \times \exp \left( -(1 - n)v - \frac{na \rho \theta}{1-\theta} v^{-1-\theta} \right) \]  

with a magnitude-dependent \(\rho\), and showed that also this form fits nicely Corral’s data including the regime of very

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that $M$ as in eq. (1), small $\tau$ marked with red boxes.

Fig.1: (Colour on-line) Interoccurrence times $\tau_i$ between events with magnitudes $M \geq 2$ from a) the SSEC and b) the NCSN catalogs. The “stationary” parts considered in this paper are marked with red boxes.

small $v$ values, for $\alpha = 0.9$, $\theta = 0.03$, $a = 0.76$ and $\rho$ such that $\rho^3 \approx 1$.

Figure 1 shows the interoccurrence times $\tau_i$ between events with magnitudes $M \geq 2$ from the two earthquake catalogs we are interested in (SSEC and NCSN [28]). In both Californian catalogs exist mainshocks characterized by large aftershock sequences with small interoccurrence times. In this letter, we focus on the stationary parts of both catalogs, where these large aftershock sequences are missing (see marked areas in fig. 1), and provide evidence that there is long-term memory in the seismic activity. We show explicitly that this memory can explain the temporal fluctuations of both magnitudes and interoccurrence times as well as, without any fit parameter, the observed form of $f(v)$ in the whole $v$-regime.

Long-term correlations. – In general, long-term persistent records $\{x_i\}, i=1, \ldots, N$, with zero mean and unit variance are characterized by an auto-correlation function $C(s) = \langle x_i x_{i+s} \rangle = \frac{1}{N-s} \sum_{i=1}^{N-s} x_i x_{i+s}$ that decays by a power law, $C(s) \sim s^{-\gamma}$, where the correlation exponent $\gamma$ is between 0 and 1; exponents $\gamma \geq 1$ describe short-term correlations and will not be considered here. The pdf of the interoccurrence times separating events above a threshold $M$, scales with the mean value $\tau_M$ as in eq. (1), i.e. $P_M(\tau) = (1/\tau_M) g(\tau/\tau_M)$ [29,30]. For large values of $v \equiv \tau/\tau_M$, the scaling function $g(v)$ is a stretched exponential, $\ln g(v) \sim -v^\gamma$, with $\gamma$ being the correlation exponent. For $v < 1$, one has approximately $g(v) \sim v^{-(1-\gamma)}$. The shape of $g(v)$ depends slightly on the distribution of the original data set. It has been found that also the interoccurrence times are long-term correlated, with the same exponent $\gamma$ as the original record which leads to a clustering of extreme events [29]. To observe these features, one needs to consider very long data sets. Otherwise, finite-size effects occur that yield an effectively larger $\gamma$ exponent. The finite-size effects are larger for exponentially distributed data than for Gaussian data [30] and may even screen the stretched exponential and the long-term memory of the interoccurrence times in short data sets. Therefore, when searching for long-term memory in the relatively short stationary seismic records, we do not try to identify these characteristic features, but instead compare the results for the observed data sets with those predicted by synthetic long-term correlated data sets of the same length and the same distribution.

To construct the synthetic catalogs we have generated Gutenberg-Richter distributed magnitudes above $M = -1$ and arranged them temporally in a long-term correlated way using Fourier filtering in combination with the Schreiber-Schmitz method (see, e.g., [2,30–32]). The occurrence times of the events above $M = -1$ can either be chosen randomly (by a Poisson process) or equidistant. Our results do not depend on this choice, since the mean interoccurrence times between events $M \geq 2$ ($M \geq 3$) are $10^3$ ($10^4$) times larger than the mean interoccurrence time between events $M \geq -1$. The lengths of the synthetic data sets were chosen such that the number of events above $M = 2$ and $M = 3$ are roughly the same as for the considered stationary parts in the two Californian catalogs (see fig. 1).

Results. – For testing the real and the synthetic records for long-term correlations we have employed the first two orders of the detrended fluctuation analysis DFA0 and DFA1 [18,33,34]. In DFA0 (DFA1), one considers the cumulated sum $Y_i = \sum_{j=1}^{i} x_j$ and studies in time windows of length $s$, the mean fluctuation $F(s)$ of $Y_i$ around the best constant (linear) fit. For long-term correlated records, $F(s)$ scales as $F(s) \sim s^{\gamma}$ with $\alpha = 1 - \gamma/2$ ($0 < \gamma < 1$), while for short-term correlated or uncorrelated records ($\gamma > 1$) $F(s)$ scales asymptotically as $F(s) \sim s^{1/2}$.

In fig. 2 we compare a part of the SSEC record with a part of the corresponding synthetic record with $\gamma = 0.4$. Figures 2a, c and d show the same part of the synthetic data set for $M \geq -1$, $M \geq 2$ and $M \geq 3$. A subsequence of this part for $M \geq -1$ is shown in fig. 2b where the characteristic mountain-valley structure of long-term correlated data can be clearly observed. For $M \geq 2$ this structure becomes less pronounced, and is hardly seen for $M \geq 3$. Of course, the long-term correlations are also present in the subsets $M \geq 2$ or $M \geq 3$, but seem to play a less dominant role. Figures 2e and f show a part of the seismic record for $M \geq 2$ and $M \geq 3$. The seismic data look quite similar to the synthetic ones. We will show in the following that the similarity is not only qualitative (as in fig. 2) but also quantitative. To this end, we have considered the stationary regimes in the Californian catalogs from fig. 1 and analyzed i) the records $\{M_i\}$ of consecutive magnitudes above $M = 2$ and above $M = 3$ and ii) the corresponding records $\{\tau_i\}$ of consecutive interoccurrence times between events above $M = 2$ and above $M = 3$, for both synthetic ($\gamma = 0.4$) and observed records.
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Fig. 2: (Colour on-line) a)–d) Sequence of artificial Gutenberg-Richter distributed correlated data with $\alpha = 0.8$. The magnitudes above $M = -1$ (a, b), $M = 2$ (c) and $M = 3$ (d) are shown. In panel b), where only a small sector of the sequence a) is displayed, one can clearly see the mountain-valley structure due to the long-term correlations and the related clustering of the events. e), f) Sequence of real earthquake data (SCEC) with magnitudes above $M = 2$ (e) and $M = 3$ (f). One can also recognize here clustering of the events. In c), e) as well as in d), f), the number of data are roughly the same.

Fig. 3: (Colour on-line) Detrended fluctuation analysis of the magnitudes from the SCEC catalog (1995–1998) (black circles), the NCSN catalog (1995–1998) (red squares) and an artificial Gutenberg-Richter distributed correlated data set with $\alpha = 0.8$ (blue triangles) for a) $M \geq 2$ and b) $M \geq 3$. The open symbols show the DFA0 results and the filled symbols the DFA1 results. Both DFA0 and DFA1 show an exponent $\alpha = 0.8$ for $M \geq 2$ and $0.70 \pm 0.10$ for $M \geq 3$, the corresponding slopes are represented by straight lines in the figure. The offsets have been introduced to separate the curves.

Fig. 4: (Colour on-line) Detrended fluctuation analysis of the interoccurrence times from the SCEC catalog (1995–1998) (black circles), the NCSN catalog (1995–1998) (red squares) and an artificial Gutenberg-Richter distributed correlated data set with $\alpha = 0.8$ (blue triangles) for a) $M \geq 2$ and b) $M \geq 3$. The open symbols show the DFA0 results and the filled symbols the DFA1 results. Both DFA0 and DFA1 show an exponent $\alpha = 0.78 \pm 0.05$ for $M \geq 2$ and $0.70 \pm 0.10$ for $M \geq 3$, the corresponding slopes are represented by straight lines in the figure. The offsets have been introduced to separate the curves.

Figure 3 shows representative results of the magnitude fluctuation functions $F_M(s)$ obtained by DFA0 and DFA1 for a) $M \geq 2$ and b) $M \geq 3$ for the two longest stationary regimes (1995–1998) of both catalogs. In both cases, the fluctuation functions in the double logarithmic plot are approximately straight lines. For both synthetic and observed records, the slopes are $0.59 \pm 0.05$ for $M \geq 2$ and $0.50 \pm 0.10$ for $M \geq 3$. We have also calculated $F_M(s)$ for $M \geq 2.5$. In this case the slope is $0.56 \pm 0.07$. Accordingly, synthetic and observed records show the same quantitative correlation behavior. The fact that in the synthetic record the effective exponents are smaller than the generated one ($\alpha = 0.8$) can be understood as follows: Consecutive magnitudes $M_i$ and $M_{i+1}$ in the subrecords $\{M_i\}$ defined above are separated in the original data set by time intervals that vary considerably around the mean interoccurrence time. These additional fluctuations play the same role as additional white noise in the $\{M_i\}$ subrecords: They weaken the correlations and thus lead to an effectively smaller DFA exponent. This effect increases with increasing $M$, and explains why for $M \geq 3$ long-term correlations even seem to be absent in the synthetic data set.

We like to stress that these results also can explain why long-term correlations in the magnitudes could not be detected in [11], where only events separated by more than 30 min have been considered. Since by this procedure the strongest correlated events have been omitted, the remaining magnitudes appear to be uncorrelated in the relatively short data sets.

Figure 4 shows, for the same data sets as in fig. 3, the interoccurrence time fluctuation functions $F_T(s)$ obtained by DFA0 and DFA1 for a) $M \geq 2$ and b) $M \geq 3$. As in fig. 3, the fluctuation functions in the double logarithmic plot are approximately straight lines, but the slopes are larger than for the magnitudes, being $0.78 \pm 0.05$ for $M = 2$ and $0.70 \pm 0.10$ for $M = 3$, for both synthetic
Table 1: Summary of the exponents \( \alpha \) of the fluctuation functions \( F_M(s) \) and \( F_\tau(s) \) obtained by DFA0 (\( F_M0 \) and \( F_\tau0 \)) and DFA1 (\( F_M1 \) and \( F_\tau1 \)) for the considered stationary regimes in the SCEC and NCSN catalogs. The error bars for the exponents depend on the length of the dataset and are between 0.05 for \( M \geq 2, 4y \) and 0.10 for \( M \geq 3, 1y \).

<table>
<thead>
<tr>
<th>Data set</th>
<th>( M \geq 2 )</th>
<th>( M \geq 3 )</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>( F_M0 )</td>
<td>( F_M1 )</td>
</tr>
<tr>
<td>NCSN 95–98 (4y)</td>
<td>0.59</td>
<td>0.59</td>
</tr>
<tr>
<td>SCEC 95–98 (4y)</td>
<td>0.59</td>
<td>0.59</td>
</tr>
<tr>
<td>NCSN 00–02 (3y)</td>
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<td>0.60</td>
</tr>
<tr>
<td>NCSN 90–91 (2y)</td>
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<td>0.61</td>
</tr>
<tr>
<td>NCSN 88–89 (2y)</td>
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<td>0.58</td>
</tr>
<tr>
<td>NCSN 88 (1y)</td>
<td>0.57</td>
<td>0.57</td>
</tr>
<tr>
<td>SCEC 84 (1y)</td>
<td>0.59</td>
<td>0.58</td>
</tr>
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and seismic records. We have also calculated \( F_\tau(s) \) for \( M = 2.5 \). In this case the slope is \( 0.75 \pm 0.07 \), for both records. The fact that in the synthetic data the observed exponents are smaller than the generated ones (\( \alpha = 0.8 \)) is, according to [30], a finite-size effect.

The results for the other stationary parts of both catalogs are in substantial agreement with figs. 3 and 4. The exponents \( \alpha \) obtained from a best power law fit are listed in table 1. In order to exclude the possibility that the long-term correlations in the seismic data are triggered by “missing data” after large magnitudes (see, e.g., [35]), we also studied a synthetic catalog created by the ETAS model (with uncorrelated magnitudes) where data have been removed according to the description of Helmstetter et al. [35]. We found that the missing data had no effect on the DFA fluctuation function of the magnitudes, \( F_M(s) \), which still scales as \( F_M(s) \sim s^{0.5} \).

Finally, we compare the pdfs of the interoccurrence times of the 7 observed records with the corresponding pdfs of the synthetic records (one of the length 1 y and one of the length 4 y). After having identified the correlation exponent \( \gamma = 0.4 \) from figs. 3 and 4 there is no free parameter to fit the pdf, since both length and distribution of the synthetic data sets are the same as those of the observed data. Figure 5a shows excellent agreement between the pdfs when plotted in scaled form, with the characteristic power law decay \( \tau^{-(1-\gamma)} \sim \tau^{-0.6} \) for \( \tau \) well below \( \tau_M \) and a stretched exponential for \( \tau \) above \( \tau_M \) [30]. For large values of \( \tau/\tau_M \) the stretched exponential is screened due to finite-size effects [30]. Figure 5b compares the pdfs of the synthetic (4 y) record from fig. 5a with the pdfs for a large number of further earthquake data obtained by Corral [22]. Also here the agreement between observed and synthetic records is excellent. As found earlier [30], the long-term correlated synthetic data show an excellent scaling over a large range of interoccurrence times, but at small ratios of \( \tau/\tau_M \) the data scatter strongly. It is remarkable that also in the observed data this feature can be seen.

**Conclusion.** – In summary, our results suggest that there exist long-term correlations between seismic events in the Californian catalogs, which show up in characteristic fluctuations in both magnitudes and interoccurrence times. These long-term correlations can also explain a part of the clustering found in Livina et al. [36]. We consider it particularly interesting that they also can explain, without any fit parameter, the
scaling form of the distribution function of the interoccurrence times in the seismic records, recently obtained by Corral [22]. In a future publication we will also study further local networks as well as worldwide seismicity.

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