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Network Reliability Analysis based on Percolation Theory

Daqing Li[1*], Qiong Zhang[1], Enrico Zio[2,3], Shlomo Havlin[4] , Rui Kang[1]

1. School of Reliability and Systems Engineering, Beihang University, Beijing, China

2. Chair on Systems Science and the Energetic challenge, European Foundation for New Energy-Electricite' de France, Ecole Centrale Paris and Supelec, France

3. Politecnico di Milano, Milano, Italy

4. Department of Physics, Bar Ilan University, Ramat-Gan, Israel

*Email: li.daqing.biu@gmail.com

Abstract

In this paper, we propose a new way of looking at the failure process of a network using percolation theory. In this new view, a network failure can be regarded as a percolation process and the critical threshold of percolation can be used as network failure criterion linked to the operational settings under control. To demonstrate our approach, we consider both random network models and real networks with different nodes and/or edges lifetime distributions. We study numerically and theoretically the network reliability and find that the network reliability can be solved as a voting system with threshold given by percolation theory. Then we find that the average lifetime of random network increases linearly with the average lifetime of its nodes with uniform life distributions. Furthermore, the average lifetime of the network becomes saturated when system size is increased. Finally, we demonstrate our method on the transmission network system of IEEE 14 bus.

Highlights

1. Statistical descriptions of the network failure comply well with practical requirements for the analysis of network reliability.
2. The percolation threshold naturally gives a network failure criterion.
3. The approach based on percolation theory is suited for calculations of large-scale networks.

Keywords

Network reliability, percolation theory, phase transition, criticality, random network

Notation

V	a set of vertices
E	a set of arcs
$\Gamma\ (V, E)$	a network defined as an undirected graph with V, E
N	the total number of nodes in a network
C_N^i	the binomial coefficient
$<a>$	the average value of the random variable a
p	the probability that a node/edge is functional
p_c	the percolation threshold
T_s	the average lifetime of the network
$a*b$	product of a and b

$[a]$ the largest integer less than or equal to a

1. Introduction

In modern society, technological networks are pervasive as they provide essential services including materials [1-2], energy [3-4], information [5] and even social communication [6]. It is not surprising, then, that network reliability is receiving particular attention, on one side as a value requested by the users and on the other side as a challenge for the service providers and network operators. One way to address the problem is to consider the connectivity structure of the network as a graph $\Gamma(V, E)$ consisting of a vertex set $V = \{v_1, v_2, ..., v_n\}$ and an arc set $E = \{e_1, e_2, ..., e_m\}$. Within this abstraction, terminal reliability can be defined as the probability of achieving connectivity from the source nodes to the terminal nodes [7]. The terminal reliability of networks can be characterized by assessment methodologies [8] such as Reliability Block Diagram (RBD), Fault Tree Analysis (FTA) [9] and so on. Typical algorithms for computing terminal reliability include the state enumeration method [10], sum of disjoint products method [11], factorization method [12], minimal cuts method [13] and cellular automata [14-15].

However, in the consideration of terminal connectivity, the identification of the operational limits of a network is missing [8], where a critical fraction of functional components to sustain the network is considered instead of studying paths in the terminal reliability. Percolation theory [16-17] provides us with an opportunity to overcome this gap, by referring network failure to the situation whereby a critical fraction of network components have failed [18-20]. In the percolation theory, the failure of a node/edge of network is modeled by removal. As the removal of nodes/edges increases, the network undergoes a transition from the phase of connectivity (functional network) to the phase of dis-connectivity (nonfunctional network). The probability threshold signifying this phase transition can be found theoretically or computed numerically by percolation theory. The probability threshold can be used as a statistical indicator for the operational limits of the network, which is not considered in traditional terminal reliability analysis. Thus, percolation theory, based on statistical physics, can help to understand the macroscopic failure behavior of networks in relation to the microscopic states of the network components. It can address questions of practical interest such as "how many failed nodes/edges will break down the whole network?"

In this paper, we define "network reliability" by using concepts of percolation theory and exploit the related statistical physics techniques to calculate it. We analyze the network failure process and network reliability properties by percolation theory, providing a new framework for network reliability analysis. In section 2, we further explain the operational limits of a network. In section 3, we relate the network reliability problem to percolation theory. In section 4, we analyze theoretically the network reliability and lifetime distribution, referring to random networks. In section 5, we present simulation results, which are extended to real networks in section 6.

To accompany the reader throughout the study of the paper, we anticipate here a number of definitions:

Definition 1 Random Network (Erdös-Rényi (ER) Network [21]): A graph with N vertices can have C_N^2 pairs at most. To generate a random network, we first build N nodes. Then we connect each pair of nodes with the same probability, p. In this way, a random network (N, p) can be constructed finally and the networks will become more connected with increasing p. Figure 1 gives an example of random network with N = 150, p = 1/70.

Definition 2 Degree of node i, k_i: the number of links that belong to node i. $<k>$: the average value of the degree, which is the sum of node's degree divided by the number of nodes in the network. In figure 1, the average degree of the network $<k> = 2$.

Definition 3 Cluster: the connected set of nodes, within which there is a path between any pair of nodes. G represents the size of the largest (giant) cluster in the network, while SG represents the size of the second largest cluster. In figure 1, G is the cluster consisting of the red nodes, and SG is the cluster consisting of the blue nodes.

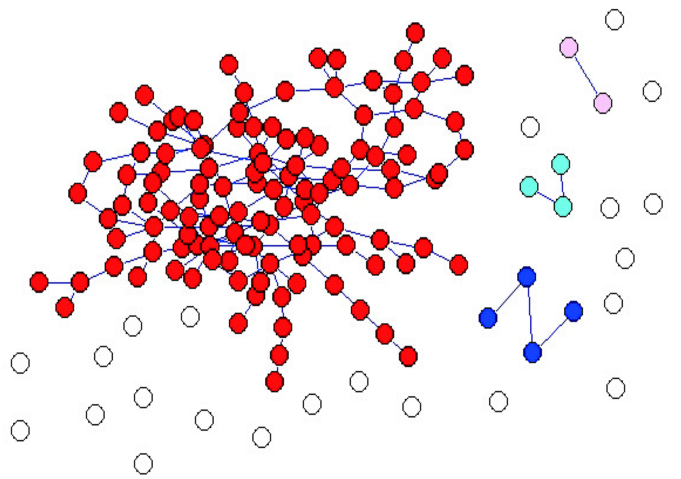


Figure 1: Random Network with $N = 150$, $<k> = 2$. G is the cluster consisting of the red nodes, and SG is the cluster consisting of the blue nodes.

2. The operational limits of a network

Given a network including communication networks and power grid, many studies focused on the terminal reliability between pair of nodes in the network. The terminal reliability includes the two-terminal reliability, K-terminal reliability and all-terminal reliability. These studies investigate the connectivity between the origin and destination of a given pair from the viewpoint of network users.

However, system operators cannot put all of the weight onto the service quality of a single user or a portion of users, and rather care about questions of practical interest such as "how many failed nodes/edges will break down the whole network?" Accordingly, we investigate the macroscopic status of network reliability by defining "the operational limits of a network" in this manuscript. When the classical methods of terminal reliability are implemented to answer above questions, the "combinatorial explosion" problem [22-23] usually occurs when the number of possible system states increases exponentially or even faster with the system size. In an attempt to overcome the "combinatorial explosion" problem in association with the assessment of network reliability, we consider the phase transition threshold of the network failure process as an indicator of network connectivity loss, and use percolation theory to identify the critical point and calculate reliability indexes correspondingly defined. Instead of focusing on verifying the existence of paths connecting source and target nodes, we study the network reliability in a system view.

Percolation theory has been widely applied in the field of complex networks [18-20]. Based on this, many studies have allowed revealing important network characteristics, including vulnerability analysis of different types of complex networks. Percolation actually describes a phase transition process of network failure, whose critical point distinguishes the network from connected to disconnected. Percolation theory makes use of statistical physics principles and graph theory to analyze such change in the structure of a complex network. Specific examples of problems, which can be described and analyzed by percolation theory, are the robustness of networks against random failures and targeted attacks [24-25], the interdependent systems [26], the spreading of infectious diseases [27].

3. Network reliability analysis based on percolation theory

In the following, by taking into account the lifetime of the network nodes, we study how the global network connectivity changes during a process of nodes and/or edges failure and measure the network reliability $R_s(t)$ and lifetime distribution $f_s(t)$ as defined with respect to the critical point of the network percolation process. Let $R(t)$ be the probability that a node/edge is functional at a given time t, i.e. the node/edge reliability at time t. A fraction $1\text{-}p = 1\text{-}R(t)$ of nodes/edges will fail according to their reliability and as the failure process proceeds, clusters of connected nodes form as they are cut off from the main (giant) network cluster (whose size is indicated as G). Then, as further nodes/edges fail, the network gradually fragments into many finite clusters. If $R(t)$ is below a critical value p_c, a main network cluster does not exist anymore and only small isolated clusters exit: we define the instant at which this occurs as the lifetime of the network. With the network topology information, this critical value p_c can be calculated according to percolation theory [18-20], which distinguishes the network from being connected to disconnected.

Compared to traditional network reliability methods, our proposed method based on the concept of percolation theory has the following advantages:

1. Statistical descriptions of the network failure comply well with practical requirements for the analysis of network reliability [28-31]. Indeed, traditional network reliability analysis mainly focuses on the effect of single paths of connection on the reliability of the network, as framed in the classical 2-end reliability, k-end reliability and full-end reliability problems. In practice, the problem of how many nodes/edges failed will break down the network system as a whole is a relevant one and its solution requires a change in the paradigm of analysis, which can be successfully tackled by percolation theory.
2. The percolation threshold naturally gives a network failure criterion. In order to identify the operational limits of a network, which is missing before in traditional reliability analysis [8], this percolation threshold can be taken as a statistical indicator of the operational limits [17]. And the research on connected cluster instead of paths during the network failure process will help to understand and analyze the microscopic origin of macroscopic network failure behaviors in a system point view.
3. The "combinatorial explosion" problem arising in the classical calculations of network reliability, in which the computational complexity grows exponentially with the number of nodes [22-23], is circumvented as the complexity in our proposed method for determining network reliability grows with the square of the network size N^2. The approach

based on percolation theory is suitable for calculations of large-scale networks.

4. Analysis of the reliability and lifetime of random networks

In this section, we will illustrate how to calculate theoretically the network reliability and lifetime based on percolation network. For a random network with N nodes, we look for the critical point p_c of the percolation process to identify the condition for the collapse of the network. As the node percolation and edge percolation have similar characteristics [16-17], the framework here will only consider node percolation for simplicity. For a small fraction 1- $R(t)$ of failed nodes (p_c<< $R(t)$<<1), only small clusters break from the giant network cluster. When the number of failed nodes increases ($R(t)$ is closer to p_c), the size of the fragments that fall off from the main cluster increases. At the critical threshold p_c, the system falls apart in the sense that the main network cluster fragments into small pieces [17]. According to percolation theory, the network loses its connectivity when the number of failed nodes reaches N-[N*p_c]. This feature enables us to employ a model of voting system [32] with threshold ([N*p_c]+1) to assess the reliability of the network. The voting system aggregates the probabilities of scenarios in which the number of nodes still functioning is greater than [N*p_c], the expected number of functional nodes to keep the network connected. Accordingly, the network reliability at time t, $R_s(t)$, can be given by the voting system based on the percolation critical value:

$$R_s(t) = \sum_{i=[N*p_c]+1}^{N} C_N^i R\ (t)^i (1-R\ (t))^{N-i} \tag{1}$$

where $R\ (t)$ is the reliability of the generic node, assumed the same for all nodes. N is the number of nodes in the network. C_N^i is the binomial coefficient.

We can derive the lifetime distribution of the network based on its reliability $R_s(t)$:

$$\begin{aligned} f_s(t) &= \frac{d(1\text{-}R_s(t))}{dt} \\ &= \frac{N!}{(N-[N*p_c]-1)!([N*p_c])!}(1-R(t))^{N-[N*p_c]-1}(R(t))^{[N*p_c]} f(t) \end{aligned} \tag{2}$$

where $f(t) = \dfrac{d(1\text{-}R(t))}{dt}$ is the lifetime distribution of the generic node.

$$T_s = \int_0^{\infty} t * f_s(t)dt = \int_0^{\infty} R_s(t)dt \tag{3}$$

The average network lifetime T_s is the time instant when a fraction of $1\text{-}p_c$ of the network nodes fail. The probability that a node is functioning at network lifetime, p_c, equals the node reliability $R(T_s)$ at the lifetime T_s of the network . So T_s can be obtained by the equation:

$$p_c = R(T_s) \tag{4}$$

For a random network, the percolation threshold can be calculated as $p_c = \dfrac{1}{<k>}$ [17].

In the next section, we consider networks with different node lifetime distributions for example:

1. Exponential distributions ($R(t) = \exp(\text{-}\lambda * t)$), where λ is the scale parameter.

2. Uniform distributions ($R(t)=\dfrac{b-t}{b-a}$), where a, b are the lower and upper limits of the interval, respectively.

3. Weibull distributions ($R(t)=\mathrm{e}^{-(t/\lambda)^k}$), where λ, k are the scale parameter and shape parameter, respectively.

5. Numerical results of random networks

We have presented the theory for network reliability based on percolation in the previous section. To demonstrate the analysis, simulation results are obtained in the following sections, which is compared with the theoretical results derived from the previous section. We generate random networks and assume that the lifetime of the nodes follows the distributions given above. For random networks, at threshold, the size of the second largest cluster, *SG*, reaches maximum, according to percolation theory [17]. Therefore, *SG* can be used as the failure indicator for random networks. We use this feature in the simulation (see figure 2 for the procedure) to identify the lifetime of the network.

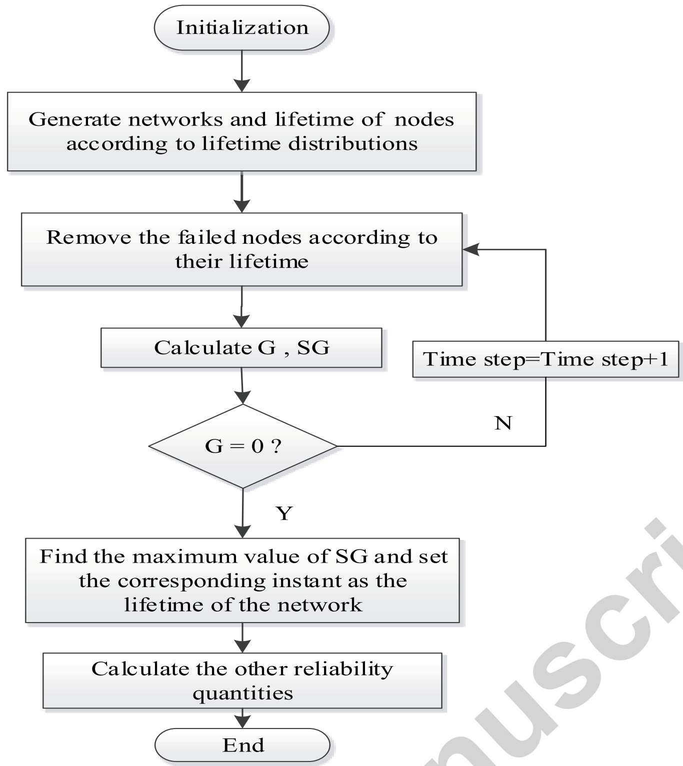


Figure 2: Simulation flowchart. G is the size of the giant cluster, SG is the size of the second largest cluster. The complexity in our proposed method for determining network reliability grows with the square of the network size N^2.

In figure 3, we present the results of our study on the reliability of random networks by analytical and simulation approaches. Three types of lifetime distributions of network nodes have been examined: exponential, uniform and Weibull. The abrupt decreases of the reliability $R_s(t)$ at the critical threshold are shown, as estimated by the analytical result of Eq. (1). The corresponding results of network reliability found show different behaviors, due to the differences in the lifetime distributions of the nodes. In figure 3a, for the random network, we assign the exponential distribution as the lifetime distribution of the network node. When time increases, some nodes begin to fail due to their limited lifetime and the network reliability begins to decrease slowly. As time further increases ($t > 5$) and more nodes become failed, the network reliability significantly decreases because the fraction of failed nodes approaches the percolation threshold. While the network reliability decreases comparatively smoothly for exponential lifetime distribution, it decreases abruptly for uniform life distribution. This is because the fraction of failed nodes increases in different way for different lifetime distributions. This may eventually influence the network lifetime, as shown in figure 3a-3c. The theoretical results are found to coincide well with the simulation results.

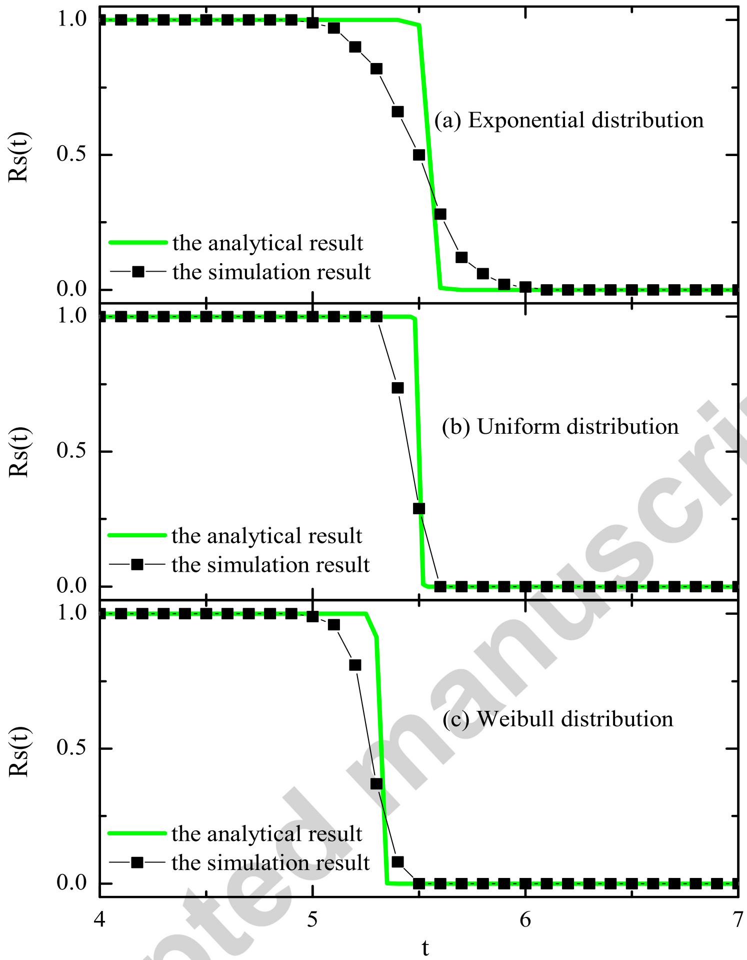


Figure 3: Reliability *Rs(t)* of a random network with <*k*> = 4, *N* = 100000 with different lifetime distributions. All of the lifetime distributions have the same average lifetime value of 4. Here *t* represents the simulation iteration in an arbitrary unit. The dotted line gives the simulation result of the network lifetime corresponding to the maximum value of *SG* whereas the solid line is the analytical result given by Eq. (1) with $p_c = 1/\langle k \rangle$ obtained by percolation theory. The difference between theory and simulation comes from the finite size effect, which decreases as network size increases. (a) Exponential distribution with λ = 1/4. (b) Uniform distribution within the range [1,7]. (c) Weibull distribution with scale parameter 4.5135 and shape parameter 2.

In figure 4, we study, by simulation, the evolution of the network topology. At $t = 0$ (in arbitrary time units), almost all nodes belong to a main cluster, and G is close to 1 while SG is approximately 0. As time goes on, G monotonically decreases while SG rises to its maximum at percolation criticality. The corresponding time is the network lifetime when the network reliability decreases almost to zero. Then, fragmentation continues until the network becomes completely disconnected. This confirms, in the case of random networks, that the maximum of *SG* can be considered as an indicator of the system breakdown and can be used to calculate the lifetime of the network. In coincidence with figure 3, as time increases, the failure of nodes decreases the size of the giant cluster *G*, due to the loss of their connections to other nodes. There are more clusters disconnecting from the giant cluster as a result of connection loss between them. As the second largest cluster *SG* reaches maximum, it indicates that the giant cluster of the network is totally fragmented. As shown in figure 4a-4c, different node lifetime distribution can influence the instance when the second largest cluster reaches maximum.

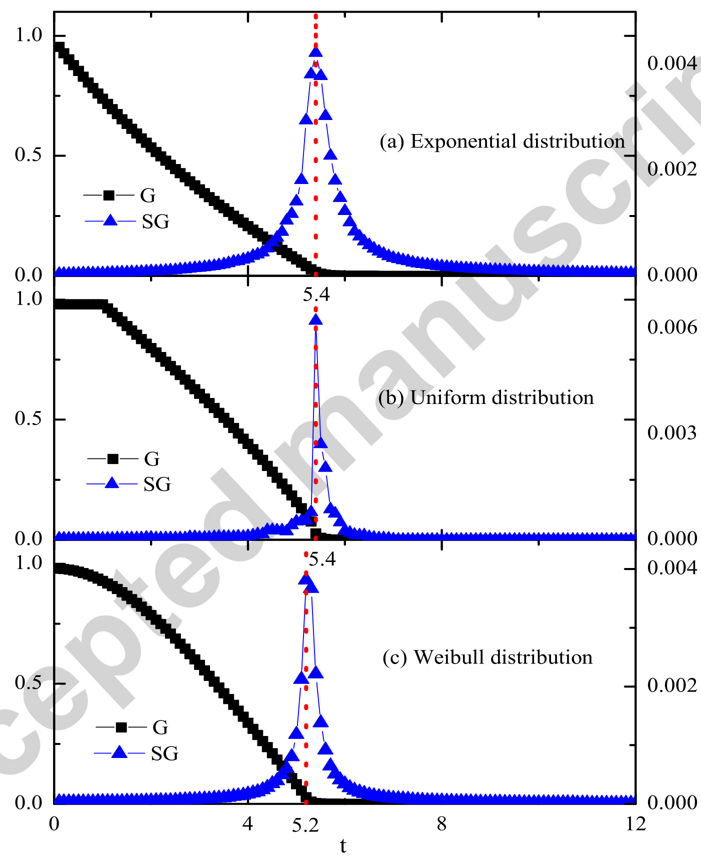


Figure 4: The evolutions of *G* and *SG* with different lifetime distributions of the nodes, where the peak of *SG* signals the network collapse. The parameters of the distributions are the same as in figure 3. (a) Exponential distribution with λ=1/4. (b) Uniform distribution within the range [1,7]. (c) Weibull distribution with scale parameter 4.5135 and shape parameter 2. The dotted lines in red mark the network average lifetime obtained by simulation.

We have also tested the relationship between the average lifetime of the network and the average lifetime of its nodes (see figure 5). When the lifetime distribution of the nodes follows the uniform distribution $f(t) = 1/(\text{b-a})$, where *a, b* are the lower and upper limits of the interval, respectively, the average node lifetime value T is $(a+b)/2$ and its reliability is $R(t)=\frac{b-t}{b-a}$. Using Eq. (4) with the percolation theory result $p_c=1/\langle k \rangle$, we get the relation between the network lifetime and the node average lifetime T:

$$T_s = T + \frac{b-a}{2} - \frac{b-a}{\langle k \rangle} \tag{5}$$

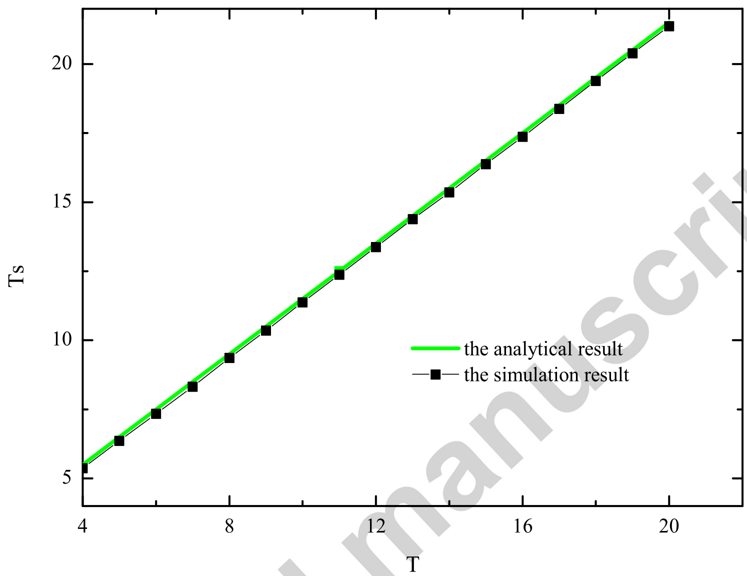


Figure 5: The average lifetime T_s of the random network changes as a linear function of the average lifetime T of the nodes (network with $\langle k \rangle = 4$, $N = 10000$). The node lifetime is assumed to obey a uniform distribution. The dotted line is the simulation result giving the network lifetime in correspondence of the maximum value of *SG* and the solid line is the analytical result from Eq.(5), with $p_c = 1/\langle k \rangle$.

Next we study the relation between the lifetime and the number of nodes N of the network; the results are reported in figure 6 for two cases:

- assuming that the lifetime distribution of all network nodes is exponential ($R(t)=\exp(-\lambda * t)$), with same average lifetime $1/\lambda$. Then, the network average lifetime is $T_s = \int_0^\infty R_s(t)dt = \sum_{i=[N*p_c]+1}^{N} \frac{1}{i\lambda}$.
- assuming that the lifetime distribution of all network nodes is uniform ($R(t)=\frac{b-t}{b-a}$, where a, b are the lower and upper limits of the interval, respectively). Then, the network average lifetime is $T_s = \int_0^\infty R_s(t)dt = \frac{N-[N*p_c]}{N+1}(b-a)+a$.

The lifetime of the network obtained by simulation is found to approach the theoretical results as the number of nodes N increases. The deviation from the analytical results for small N is mainly due to the finite size effect. Figure 6 suggests that network lifetime is insensitive to the size of network in our investigated random network configurations. This is partially because the model here does not consider the cascading failures between network components. When the mechanism of cascading failures is introduced, specific study is needed to investigate the effect of network size on the network reliability.

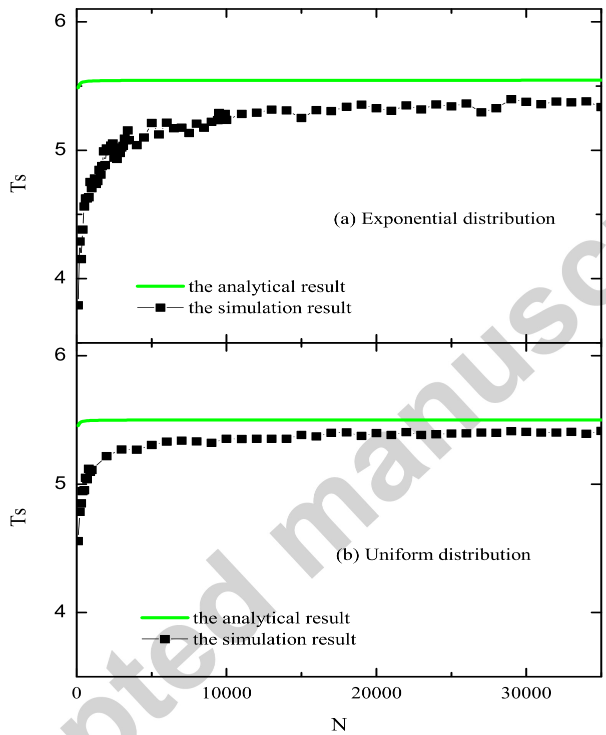


Figure 6: The average lifetime of the random network changes as a function of the network size N. (a) The node lifetime obeys an exponential distribution with $\lambda = 1/4$. (b) The node lifetime obeys a uniform distribution within the range [1,7]. The dotted line is the simulation result giving the network lifetime in correspondence of the maximum value of SG; the solid line is the analytical result with $p_c = 1/\langle k \rangle$.

6. Numerical results of a real network

In this section, we use the transmission network system IEEE 14 bus [33] as reference case study. The IEEE 14 Bus Test Case represents a portion of the American Electric Power Grid (in the Midwestern US), which is widely implemented for different case studies including short circuit analysis, load flow studies and interconnected grid problems. The network represents a portion of the Electric Power System with 14 bus locations connected by 20 lines and transformers. For the analysis, we refer to nodes and edges to represent the network components. In this case study, failures of edges are considered, and with different rate and probability values taken from tables 1 and 2 of reference [33]. For completeness, the failure rate data are reported in table 1. The failure probability of edge *ij* is defined as:

$$F_{ij}(t) = 1 - e^{-\lambda_{ij} t}$$

Table 1. Failure rate data of the edges in the network considered.

From vertex	To vertex	Failure rate/year
1	2	0.2389
1	5	0.8795
2	3	0.7818
2	4	0.6949
2	5	0.6841
3	4	0.6732
4	5	0.1629
4	7	0.01045
4	9	0.01045
5	6	0.01045
6	11	1.1944
6	12	1.5364
6	13	0.7818
7	8	0.01045
7	9	0.01045
9	10	0.5049
9	14	1.6233
10	11	1.1509
12	13	1.1998
13	14	2.0902

In figure 7, we present the simulation analysis of the real network reliability. Note the similarity between this figure and figure 3 for the random network model: $R_s(t)$ drops abruptly at the percolation threshold, and then decreases slowly due to the small failure rate of some edges (table 1). In IEEE 14 bus, our results in figure 7 suggest that timely maintenances should be implemented before this abrupt collapse of network.

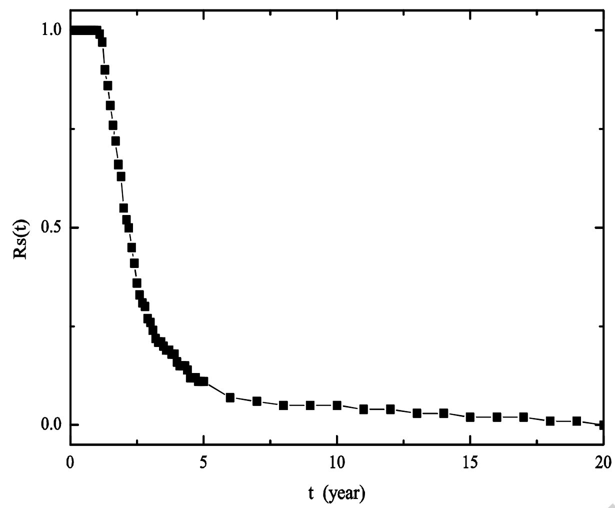
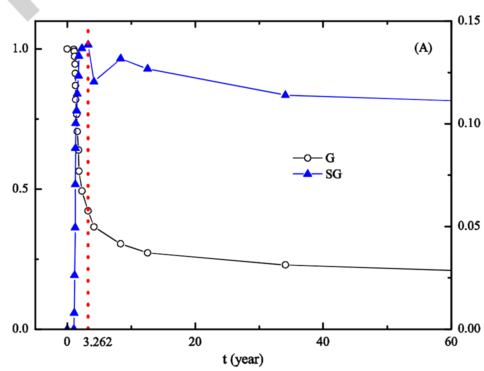


Figure 7: Reliability *Rs(t)* of the transmission network with exponential distributions of edge lifetime.

In figure 8, we study the evolution of the network topology. Again, the instant when the maximum of *SG* occurs is used to identify the breakdown of the system. Because the edge failure rates values are heterogeneous, the evolution of *SG* is not bell-shaped as in the homogeneous case (all identical values of failure rates) of figure 4. The findings in figure 8a explain the reason of abrupt decrease of network reliability found in figure 7. According to the lifetime of network component (table 1), the giant cluster is fragmented suddenly after the loss of a few components. These components are considered as the vulnerable part of the whole system. For example, the $link_{611}$ and $link_{612}$ with high failure rates play an important role in bridging two functional clusters of the transmission network. The failure of these links at the very beginning makes the network fragile, comparatively.



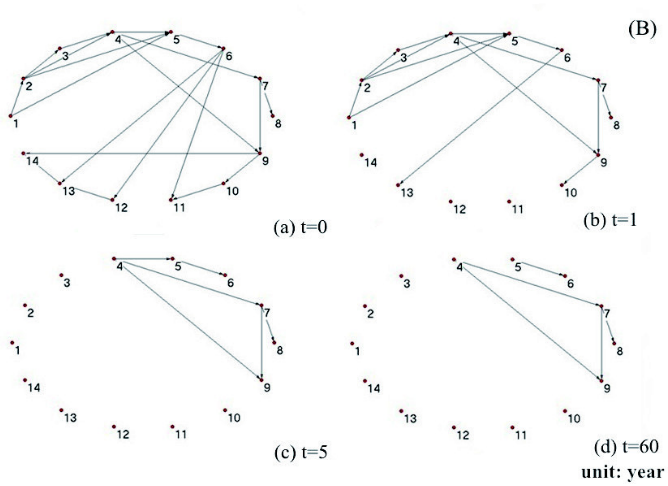


Figure 8: (a) The evolutions of *G* and *SG* of the transmission network with exponential edge lifetime distributions. The dotted line marks the average lifetime obtained by simulation. (b) The evolution of the topology of the transmission network during the failure process.

7. Conclusion

In this paper, we adopt a statistical physics description of the failure processes in a network system and build a framework based on percolation theory to calculate network reliability. Different from traditional considerations, here we study the network reliability properties using the percolation process and use the critical threshold of percolation as network failure criterion. Compared with traditional network reliability methods, the framework can give a practical understanding of network global connectivity failure instead of focusing on terminal connectivity and therefore circumvents the computational problems of classical network reliability analysis methods in large-scale networks. The framework can consider both nodes and edges failures, and with different reliability functions. Here we calculate the network reliability without considering a variety of realistic failure behaviors, including cascading failure processes in single [34-37] and interdependent networks [38-39]. These will be subjects of future investigations in the continuation of this research.

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References

[1] Wu WW, Ningb A, Ning XX. Evaluation of the reliability of transport networks based on the stochastic flow of moving objects. Reliability Engineering & System Safety 2008; 93: 838–844;

[2] Zio E, Sansavini G, Maja R, Marchionni G. An Analytical Approach to the Safety of Road Networks. Int. J. of Reliability, Quality and Safety Engineering 2008; 154:67-76.

[3] Zio E, Piccinelli R. Randomized flow model and centrality measure for electrical power transmission network analysis. Reliability Engineering & System Safety 2010; 95: 379 – 385.

[4] Lin YK, Yeh CT. Maximal network reliability for a stochastic power transmission network. Reliability Engineering & System Safety 2011; 96: 1332–1339.

[5] Rai S, Kumar A, Prasad EV. Computing terminal reliability of computer network, Reliability Engineering 1986; 16: 109-119.

[6] Schneider K, Rainwatera C, Pohl ED, Hernandez I, Emmanuel J, Marquez R. Social network analysis via multi-state reliability and conditional influence models. Reliability Engineering & System Safety 2013; 109: 99–109.

[7] Wilkov R. Analysis and Design of Reliable Computer Networks. IEEE Transactions on Communications 1972; 20: 660-678.

[8] Zio E. Reliability engineering: Old problems and new challenges. Reliability Engineering and System Safety 2008; 6: 128-133.

[9] Volkanovski A, Čepin M, Mavko B. Application of the fault tree analysis for assessment of power system reliability. Reliability Engineering & System Safety 2009; 94: 1116–1127.

[10] Shier DR. Network Reliability and Algebraic Structures. Oxford: Clarendon Press; 1991.

[11] Wilson JM. An Improved Minimizing Algorithm for Sum of Disjoint Products. IEEE Transactions on Reliability 1990; 39: 42-45.

[12] Wood RK. Factoring Algorithms for Computing K-Terminal Network Reliability. IEEE Transactions on Reliability 1986; 35: 269-278.

[13] Lin YK. Using minimal cuts to evaluate the system reliability of a stochastic-flow network with failures at nodes and arcs. Reliability Engineering & System Safety 2002; 75:41-46.

[14] Rocco CM, Zio E. Solving Advanced Network Reliability Problems by Means of Cellular Automata and Monte Carlo Sampling. Reliability Engineering and System Safety 2005; 89: 219-226.

[15] Zio E, Podofillini L, Zille V. A Combination of Monte Carlo Simulation and Cellular Automata for Computing the Availability of Complex Network Systems. Reliability Engineering and System Safety 2006; 91: 181-190.

[16] Stauffer D, Aharony A. Introduction to percolation theory. Great Britain: Taylor& Francis; 2003.

[17] Bunde A, Havlin S. Fractals and Disordered Systems. Berlin and New York: Springer-Verlag; 1994.

[18] Albert R, Barabási AL. Statistical mechanics of complex networks. Reviews of Modern Physics 2002; 74:47-97.

[19] Boccaletti S, Latora V, Moreno Y, Chavez M, Hwang DU. Complex networks: Structure and dynamics. Physics reports 2006; 424: 175-308.

[20] Cohen R, Havlin S. Complex Networks: Structure, Robustness and Function. Cambridge University Press, 2010.

[21] Erdös P, Rényi A. On the evolution of random graphs. Publ. Math. Inst. Hung. Acad. Sci 1960; 5: 17-61.

[22] Krippendorff K. Combinatorial Explosion. Web Dictionary of Cybernetics and Systems. Principia Cybernetica Web 2010.

[23] Schuster P. Taming combinatorial explosion. Proceedings of the National Academy of Sciences 2000; 97(14):7678-7680.

[24] Cohen R, Erez K, Avraham DB, Havlin S. Resilience of the Internet to Random Breakdowns. Physical Review Letters 2000; 85: 4626-4628.

[25] Cohen R, Erez K, Avraham DB, Havlin S. Breakdown of the Internet under Intentional Attack. Physical Review Letters 2000; 86: 3682-3685.

[26] Buldyrev SV, Parshani R, Paul G, Stanley HE, Havlin S. Catastrophic cascade of failures in interdependent networks. Nature 2010; 464: 08932.

[27] Newman MEJ. Spread of epidemic disease on networks. Physical Review E 2002; 66: 016128 (1-11).

[28] Mu DZ, Qiu ZD. On the connectivity and reliability of the large-scale wireless sensor network with the percolation. IET International Conference on 2006;1: 1-3.

[29] Kong ZN, Yeh EM. Correlated and cascading node failures in random geometric networks: A percolation view. Ubiquitous and Future Networks (ICUFN), 2012: 520-525.

[30] Pennetta C, Reggiani L, Trefan G. A percolative approach to reliability of thin films. IEEE Transactions on 2000; 47:1986-1991.

[31] Raman V, Gupta I. Performance Tradeoffs among Percolation-Based Broadcast Protocols in Wireless Sensor Networks. Distributed Computing Systems Workshops.2009:158-165.

[32] Elsayed AE. Reliability Engineering. Wiley, 2012:154-155.

[33] Zio E, Piccinelli R, Delfanti M, Olivieri V, Pozzi M. Application of the load flow and random flow models for the analysis of power transmission networks. Reliability Engineering and System Safety 2012; 103:102–109.

[34] Zio E, Sansavini G. Component Criticality in Failure Cascade Processes of Network Systems. Risk Analysis 2011; 31: 1196-1210.

[35] Sansavini G, Hajj MR, Puri IK, Zio E. A Deterministic Representation of Cascade Spreading in Complex Networks. Europhysics Letters 2009; 87: 48004.

[36] Liu CR, Li DQ, Fu BW, Yang SK, Yunpeng Wang, Guangquan Lu. Self-healing from cascading failures in complex networks. EPL 2014; 107: 68003.

[37] Li DQ, Jiang YN, Kang R, Havlin S. Propagation behavior of cascading overload failures. Nature Scientific Reports 2014; 4:5381.

[38] Zio E, Sansavini G. Modeling Interdependent Network Systems for Identifying Cascade-Safe Operating Margins. IEEE Transactions on Reliability 2011; 60: 94-101.

[39] Zio E, Ferrario E. A framework for the system-of-systems analysis of the risk for a safety-critical plant exposed to external events. Reliability Engineering & System Safety 2013; 114