Journal of Complex Networks (2020) **2**, Advance Access Publication on 31 May 2020
doi: 10.1093/comnet/cnaa013

Review

Cascading failures in complex networks

LUCAS D. VALDEZ†
Network Science Institute, Northeastern University, Boston, MA, USA
†Corresponding author. Email: ldvaldez@bu.edu

LOUIS SHEKHTMAN
Department of Physics, Bar Ilan University, Ramat Gan, Israel

CRISTIAN E. LA ROCCA
Instituto de Investigaciones Físicas de Mar del Plata (IFIMAR)-Departamento de Física, FCEyN, Universidad Nacional de Mar del Plata-CONICET, Mar del Plata, Argentina and Department of Physics, Boston University, Boston, MA, USA

XIN ZHANG
College of Transport and Communication, Shanghai Maritime University, Shanghai, China and Department of Physics, Boston University, Boston, MA, USA

SERGEY V. BULDYREV
Department of Physics, Yeshiva University, New York, New York, USA; Department of Management, Economics and Industrial Engineering, Politecnico di Milano, Milano, Italy and Department of Physics, Boston University, Boston, MA, USA

PAUL A. TRUNFIO
Department of Physics, Boston University, Boston, MA, USA

LIDIA A. BRAUNSTEIN
Instituto de Investigaciones Físicas de Mar del Plata (IFIMAR)-Departamento de Físicas, FCEyN, Universidad Nacional de Mar del Plata-CONICET, Mar del Plata, Argentina and Department of Physics, Boston University, Boston, MA, USA

AND

SHLOMO HAVLIN
Department of Physics, Bar Ilan University, Ramat Gan, Israel; Department of Physics, Boston University, Boston, MA, USA and Tokyo Institute of Technology, Yokohama, Japan

Edited by: Ernesto Estrada

[Received on 3 February 2020; editorial decision on 17 March 2020; accepted on 8 April 2020]

Cascading failure is a potentially devastating process that spreads on real-world complex networks and can impact the integrity of wide-ranging infrastructures, natural systems and societal cohesiveness. One of the essential features that create complex network vulnerability to failure propagation is the dependency among their components, exposing entire systems to significant risks from destabilizing hazards such as human attacks, natural disasters or internal breakdowns. Developing realistic models for cascading failures as well as strategies to halt and mitigate the failure propagation can point to new approaches to restoring and strengthening real-world networks. In this review, we summarize recent progress on models developed based on physics and complex network science to understand the mechanisms, dynamics and overall impact of cascading failures. We present models for cascading failures in single networks and interdependent networks and explain how different dynamic propagation mechanisms can lead to an abrupt collapse and a rich dynamic behaviour. Finally, we close the review with novel emerging strategies for containing cascades of failures and discuss open questions that remain to be addressed.

Keywords: cascading failures; complex networks; spatial networks; network of networks; percolation; network robustness

1. Introduction

It is well accepted that many complex systems can be represented, analysed and better understood as networks. Examples include World Wide Web, social and related online networks; the power grid, Internet, traffic, airline and related infrastructure networks; and neural and physiological networks [1]. Formally, a network or graph is a set of nodes or vertices connected via internal links. Despite the network's mathematical simplicity, it also establishes a powerful and versatile tool to characterize and understand many complex systems in nature, technology and society. In a node-link network structure, one of the main measures characterizing topology is the degree distribution $P(k)$ which gives the probability that a node has k connections (links). Networks with a characteristic average degree or connectivity can be regarded as homogeneous, whereas networks that lack a characteristic degree and instead consist of a broad range of degrees can be regarded as heterogeneous networks. A typical example of a homogeneous network is the Erdős–Rényi network which has a Poisson degree distribution, while for heterogeneous networks a typical example is the Scale-Free (SF) network, with a power law degree distribution with exponent λ. Another important feature of real-world networks is that the nodes are not randomly connected but display non-random patterns of connections with features such as clustering, degree-degree correlations and modularity [1, 2].

In our everyday world, a large number of processes occur on top of networks, such as the spread of diseases in contact networks, opinion formation in social networks and synchronization of neurons in the brain [1, 2]. One of the most dramatic processes that spread on complex networks is the cascade of failures when a failure in part of the system leads to further failures in the same and other systems which then continue to propagate. Eventually, the entire system could become dysfunctional and catastrophically collapse. For example, in an interdependent system such as the organs of the human body, the malfunction of the nervous system or another physiologic system can affect various organs generating negative feedback that could lead to system collapse and death. Similarly, in an electrical power grid, the damage of a node could trigger an overload on other components of the grid that further propagates to the entire system that could lead to a catastrophic collapse of the system [3–5]. The initial failure that triggers the cascade could be due to human attacks or natural disasters, such as the tsunami in Japan in 2011, and the forest fire in California in August 2018. Hurricane Katrina in 2005 is another example of extreme event with catastrophic effects on several systems [6–8]. In addition, studying cascading failures can explain why networks evolve towards topologies that attenuate these cascades. For example, for the *Escherichia coli*, Ref. [9] showed that for a gene regulatory domain (network) which depends on a metabolic domain through a protein domain, small perturbations originated in the metabolic domain trigger smaller cascades than those originated in the gene regulatory network. The higher robustness of the metabolic network is explained because it is more coupled with the environment than the other domains, and hence, the metabolic network needs to be more robust under external fluctuations. On the other hand, Ref. [10] showed for networks obtained from functional magnetic resonance imaging (fMRI) experiments and simulations, that different modules in the functional brain network have correlations in their node connectivities which increase the robustness of the system against cascading failure.

Studying cascades of failures in networks is not only critical for understanding how the network structure impacts its resilience to catastrophic cascades, but more importantly it helps to develop tools that can predict, mitigate, prevent and recover the system from such failures. Physics and network science have an essential role in understanding cascading failures since tools from statistical mechanics, such as phase transitions, percolation theory and non-linear dynamics, are useful to describe and comprehend the process, and could help in developing strategies to halt or mitigate the catastrophic effects of collapse. Here, we review the main physics models used to improve our understanding of the dynamics of cascading failures. We must emphasize that these physics models are simplified models of real infrastructures and their predictions serve to demonstrate mechanistic possible behaviours of systems, rather than provide the type of precise predictions for a particular system as would be done in engineering. Any studies of real interdependent infrastructures must take into account technical details of the system under investigation and thus must depend on a large number of parameters. This makes it very difficult to gain any mechanistic insight into the actual underlying processes leading to the observed behaviours, which is the goal of physicists.

2. Cascade of failures in single complex networks

The study of dynamic spreading models in single or isolated networks allows us to discover how different types of internal dynamics affect the system functionality or overall robustness. Dynamic processes can occur in isolated networks and propagate via cascades, that is, processes in which a component that becomes dysfunctional lead to other components that depend on it (directly or indirectly) to also become dysfunctional. The origin of this cascade has several possible sources and can, for example, be due to: (i) the redistribution of the load on a node or link leading to overloads as the model of Ref. [5], (ii) the existence of direct dependencies in which if a node becomes dysfunctional then all the nodes that depend on it also become dysfunctional [11] or (iii) the number of functional neighbours is above or below a threshold as in bootstrap and k-core percolation, respectively [12–15]. Note, that the overload models are in many ways similar to the models with direct topological dependencies: node or link A, whose failure was caused by the overload due to redistribution of load after failure of node or link B, dynamically depends on B, because A cannot function after B fails. However, the major distinction between the overload and topological models is the dependence of the former on the sequential order of failures, while for the latter the final outcome depends only on the topology of network. The vulnerability of single networks to the cascade of failures can be seen in an abrupt transition at a critical fraction of functional nodes. While the motivation of some of these models is to study failure propagation in infrastructure systems, they could be applied to other processes that propagate as cascades, such as, activation process in living neural networks in which cascades emerges because neurons fires under external stimulation or if they receive fires from their input neurons [16–18]. Another possible application is the study of information propagation in social networks, like the propagation of a hashtag or meme in Twitter, since real data suggested that individuals transmits the information as a complex contagion process where an individual needs multiple inputs

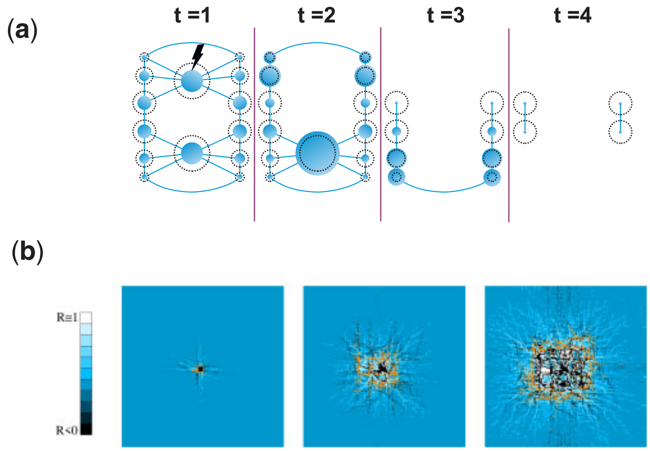

FIG. 1. (a) Schematic figure of the time evolution of the overload model presented in Ref. [5]. The length of the radius of the node (blue circle) represents its load, that is, the number of shortest paths that pass through that node, while the radius of the black dashed circle represents its capacity. In step $t = 1$, one node fails which will change the load of the rest of nodes. In the following steps, nodes fail if their load exceed their capacity and they are removed. (b) Time evolution of the propagation of the overload failures in a lattice under localized attack at the centre [21] for time $t = 2$, $t = 4$ and $t = 6$ (from left to right). The parameter R represents how much free capacity a node has available ($R \approx 1$ corresponds to a node without load, and $R < 0$ corresponds to a failed node because its load exceeded its capacity). For each snapshot at time t, the nodes that fail at that time are in orange.

from several neighbours to transmit the information [19, 20]. In this section, although not exhaustive, we present models of cascades of failures in single networks and some of the most recent insights.

A simple overload model was proposed in Ref. [5] to study the cascade of failures triggered by the initial failure of one node. In this model, each node has a fixed maximum capacity to support load which is given by node betweenness, that is, the number of shortest paths that pass through that node. This maximum capacity is defined as the initial load of the node multiplied by $(1+\alpha)$, where $\alpha > 0$ is called tolerance. If a node's load exceeds its maximal capacity, then the node fails and the shortest paths change, generating subsequent overloads and failures and until the system reaches a steady state (see Fig. 1a). For heterogeneous networks, if the initial failure is random, the system is so robust that cascades of failure usually do not arise. However, if the node with the highest connectivity or the highest load is the first to fail, then a cascade is triggered, and the size of the functional network significantly shrinks. This model was extended on embedded two-dimensional Euclidean networks to understand how a localized attack (in which a randomly chosen node and several shells around it are removed) leads to propagation of cascading failures through the network [21] (see Fig. 1b). In these networks, a universal behaviour of the failure propagation has been observed, since in different network substrates, the overload propagates at a nearly constant velocity which increases as the tolerance of the system decreases. In this research, the authors suggest that cascading overloads can be mapped into dependency cascading failures. A similar propagation has also recently been observed for the models of traffic jams [22].

Recently, the model of Ref. [5] has been explored for a case when a macroscopic fraction $1-p$ of nodes fails at the initial condition [23], as opposed to one node in the original model [5]. In this case, the order parameter can be taken either as the fraction of survived nodes, or as the fraction of nodes in the giant component (GC). At the end of the cascade, the system undergoes a first-order transition on the α, p plain for small α. However, as the tolerance increases, and p decreases the transition becomes continuous as in ordinary percolation.

Similar threshold models [4, 24, 25] have been used to simulate cascading failures in a more realistic direct current approximation of the electric power grid, in which the nodes are connected with transmission lines of given resistance and the nodes can be of three types: generators adding current to the system, loads taking current away from the system, and transmission nodes, which conserve current. Each node satisfies the Kirchhoff equation. At the beginning, a line or a fraction of lines is destroyed. As a result, the current redistributes, and if the current in a line exceeds a certain threshold, established by the $N-1$ criterion [26], the line also fails. This process leads to cascading failures, in which the distribution of the blackout sizes follows an approximate power law as supported by the empirical statistics [3]. This and earlier observations and modelling of the cascades in power grids lead in Refs [25–34] to a conjecture that the power grids drive themselves to the critical point like the self-organized criticality models [35–38].

However, similar models [39, 40] display the same phenomenology as in the model of Ref. [5] based on betweenness, showing a bimodal distribution of the blackout sizes. The reason for this difference is that these models follow different rules of cascade propagation. In the former models, at each stage of the cascade, only one of the overloaded lines is removed after which the currents in rest of the lines are recalculated, while in the latter all overloaded nodes are removed at once. This difference illustrates a crucial difference between the overload models in which the outcome depends on the dynamics of the cascade and the topological models, in which the outcome is independent of the dynamics. The seminal paper of Hines *et al.* [41] who include in their model dynamical effects also observed the difference in behaviour of cascades in topological models of failure and the models based on overloads.

The importance of transient dynamics in the cascade of failures was also demonstrated in Refs [42, 43]. In Ref. [42], it was observed that after the failure of a single link, the nodes at a distance closest to the initial failure tend to fail first, where distance is defined as the smallest weighted shortest path. This finding sheds new light on the understanding of propagation patterns on networks by measuring the speed of a cascade.

Another direction of research on cascade of failures in power grids involves identifying those variables that are good predictors for the final size of a cascade. It was found that one of the predictive factors is the vulnerability of a component, that is, the probability that it fails, and another predictive factor is the co-susceptibility which is the tendency for a group of components to fail together [44]. This phenomenon is similar to the concept of dependency groups that were modelled in single networks [11] and interdependent networks [45]. Another concept that has been proposed to study for the cascade of failures in power grids is the 'vulnerable set' which is defined as the set of lines with vulnerability above a given threshold value [46]. Using this measure and through stochastic simulations of the USA and South Canada power grid network, it was found that a small fraction of the transmission lines in the network is vulnerable to malfunction or physical damage, called 'primary failure'. Moreover, the topological and geographical distances of the initial failure to this vulnerable set determines the amount of cascading failures, as measured by the reduction in the amount of power delivered to consumers.

While many models describing the cascade of failures in single networks correspond to irreversible processes, in practice, a node can often return to its previous state. These systems could describe, for example, neural networks in which there are inhibitory and excitatory neurons leading to cascades of activation or deactivation of neurons. Introducing the concept of recovery in a model, Ref. [47] studied nodes that can fail either spontaneously or if their number of active neighbours are below a threshold, and included the ability of nodes to recover, that is, return to the active state after a period of time. They found that the fraction of active nodes exhibit hysteresis and if the system size is finite then the system flips between a state with a majority of active nodes and a state with an inactive majority. In Refs. [48, 49], it was found that this model also exhibits oscillations in the fraction of active and inactive nodes for some values of the control parameters. Similarly, for a model of neural networks in complex networks, the system can exhibit hysteresis in the fraction of active neurons, as well as oscillations even for a network with a small number of nodes [50].

In the following section, we review several studies of cascade of failures in interdependent networks showing that dependency links between networks lead to cascading failures between networks, rich dynamics and even to abrupt transitions.

3. Cascade of failures in interdependent networks

Before 2010, researchers primarily studied processes on single networks. However, real-world networks such as the infrastructure systems are not isolated but depend on one another [51, 52]. Crucially, if a small failure occurs in one system it could lead to other failures and propagate, leading to the complete collapse of the system of systems. Such systems of systems can be represented as interdependent networks composed of individual networks that depend on each other for functionality. Within each individual network, the links between nodes are *connectivity* links, while between the networks *dependency* links connect the nodes of different networks. The connectivity links represent an interaction between nodes of the same type, for instance, communication between computers or electricity flow between power towers. On the other hand, a dependency link represents an indispensable relation between two nodes, in such a way that the failure of one node implies that the other node will fail even if it is still connected to its network. In interdependent networks, the cascading failure consists of a feedback mechanism in which the failure propagates among the networks back and forth through dependency links until the system reaches the steady state.

In these interdependent networks, the functionality of nodes in one network depends on the state of the nodes in other networks. In these systems, a failure in one network may trigger a domino effect which can result in the collapse of the whole system. For example, the great blackout of Italy in 2003 demonstrated that breakdowns in power grids strongly impact other systems such as communication and transportation networks, and the failure of these networks, in turn, accelerates the failure of the power grid [53]. The modelling of the propagation of this cascade of failures across several networks has received broad interest in recent years [54–56] because many real-world systems are not isolated but depend on others. Such interdependency can make the system more vulnerable compared to single isolated networks. The manifestation of this vulnerability is seen in the propagation of failures as a cascade and the emergence of an abrupt collapse: in isolated networks, the percolation transition is a continuous second-order transition while in an interdependent system it is a hybrid phase transition [57, 58]. For interdependent systems near the transition point, the system is metastable: very small additional damage can cause the complete collapse of the system, which otherwise appears stable.

A simple model of interdependency between networks was developed by Buldyrev *et al.* in 2010 [55] and showed that such systems could undergo abrupt collapse under random failures. In this study, the system consists of two interdependent random networks A and B, in which nodes in one network depend via a one-to-one correspondence on nodes in the other network and vice versa (see Fig. 2). The functionality of the network depends on internal and external rules [59] of failure propagation. The internal rules govern the conditions that a node in a network will fail exclusively due to the states of the nodes in the same network. On the other hand, the external rules indicate under which conditions a node fails due to the states of the nodes in the other networks. The internal rule of failure for the particular process described in Ref. [55] (called mutual percolation) is that nodes that do not belong to the GC of each network fail,

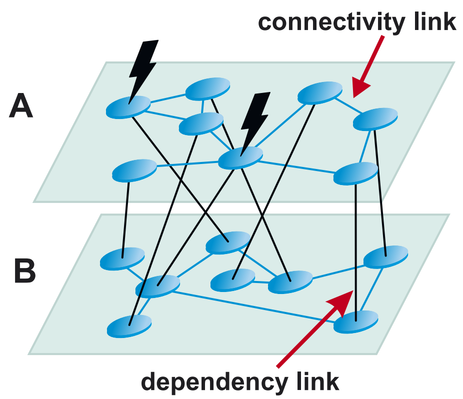

FIG. 2. Schematic representation of two interdependent networks A and B with one-to-one interdependency where two nodes in network A fail externally (represented by lightning). Blue links are *connectivity* links and black links are *dependency* links.

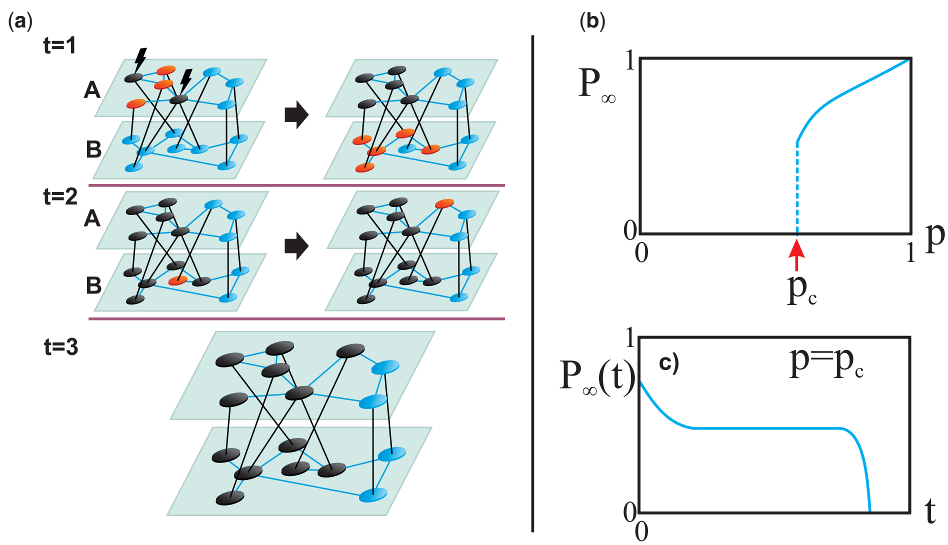

FIG. 3. (a) Schematic illustration of the temporal evolution of the cascade of failures for the model presented in Ref. [55]. The black nodes are dysfunctional, blue nodes are functional and red nodes will become dysfunctional because they belong to finite clusters or fail due to the interdependence. In step $t = 1$, on the left, two nodes fail, so network A separates into three clusters. The two smallest clusters will become dysfunctional which will cause the nodes of network B to fail in the next time step (see the interdependent network on the right for $t = 1$). At $t = 2$, the nodes of network B fail. At $t = 3$, the steady state is reached. (b) Schematic figure of the fraction of nodes in the GC, P_∞, when a fraction $1 - p$ is initially removed. The red arrow indicates the value p_c at which the system undergoes cascading failures and abrupt collapse. Although the transition is abrupt, it is different from a normal abrupt transition since just above but close to the threshold point, the fluctuations of the sizes of the GC diverges as a power law [57, 60]. (c) Schematic figure of the time evolution of the fraction of nodes in the GC, $P_\infty(t)$, at $p = p_c$. Before the system collapses, $P_\infty(t)$ has a long plateau which increases when the system size increases [55, 58].

while the external rule of failure is that if a node fails in one network, its interdependent node in the other network also fails. The process of cascade of failures begins with a random failure of a fraction of nodes in network A. This failure generates finite clusters that do not belong to the GC which fail and whose failures propagate to network B through the dependency links. Subsequently, finite clusters in network

B become dysfunctional and propagate the failure back to network *A* and so on. This cascading failures process is repeated until the system reaches a steady state in which there are no more finite clusters (see illustration in Fig. 3 a) and the sizes of both GC are the same. In the thermodynamic limit, Buldyrev *et al.* [55] showed that in a random network there exists a threshold value of the initial failure $1 - p_c$, at which the GC in both networks collapses abruptly since they cannot sustain each other (see Fig. 3b). Around the transition point, the time evolution of the size of the GC undergoes a plateau stage in which it decreases very slowly, and after that, the dilution accelerates and the system collapses (see Fig. 3c). The importance of this stage is that a useful strategy could take advantage of this slow dynamic to stop the cascade of failure using only a small number of resources.

One of the main advantages of the mutual percolation model is its simplicity which, allows it to be studied using percolation theory [55, 61, 62]. In addition, Ref. [57] showed that interdependent networks can be studied analytically in terms of avalanches. However, we must emphasize that this model cannot be directly applied to the interdependence of the power grid and Supervisory Control and Data Acquisition (SCADA) system [63] which was mentioned in Ref. [55] as an illustration only, because the internal rule of failures in the power grid is not related to the global connectivity. In fact, islanding, that is, separating the power grid into disconnected independent parts can be an efficient tool for stopping cascade propagation [40, 64]. Moreover, Ref. [55] ignores technical details of the SCADA system [63]. Following Ref. [55], several studies have incorporated more realistic features into the original model and investigated their effects on the cascade of failures, such as degree–degree correlations [60, 65–69], link overlap [70–74], clustering [75–77], directed networks [78, 79], multiple interdependent networks [80–84], multiple dependencies [59, 85], spatial embedding effects (will be discussed in the next section) [86–88, 136], hyperbolic interdependent networks [89], targeted and localized attack [90–92] and generalization of interdependence to dynamical systems [93–95]. Below, we summarize some of the lines of research that have been developed in the last years. Note that most of these extensions include the GC internal rule and the one-to-one dependency.

Multiple interdependent networks: References [82–84] generalize the mutual percolation model to the system of more than two independent networks, or Network of Networks (NON), focusing on how different topologies at the global scale (of interdependence between networks) affect the size of the GC and the percolation threshold. At a global scale, networks are represented by 'supernodes', and a 'superlink' between two supernodes represents all the dependency links between these networks. In these primitive models of NONs the dependency links establish a one-to-one correspondence (isomorphism) between the nodes of all the networks in the NON, which means that if a superlink connects two networks *A* and *B*, this implies that every node in *A* has a dependency link towards a unique node in *B* and vice versa. In subsequent works [57, 61, 81], it was explained that the mutual GC in these networks does not depend on the global topology of the NON by arguing that networks with isomorphism are equivalent to a single network with multiple layers of links (called a multiplex), regardless of the global structure (tree-like or with loops).

Autonomizing interdependent networks: In contrast to the original model of interdependent networks (Ref. [55]), it is expected that in real systems there exist nodes that are independent or autonomous since, for example, a communication system in SCADA may have battery supplies, and hence it does not depend on any node of the power grid. Reference [96] studied the effect of random autonomization in which a fraction $1 - q$ of randomly chosen nodes do not depend on the nodes of the other network. In the limit $q = 1$, the networks are fully interdependent as in Ref. [55] and the system experiences an abrupt collapse, while for $q = 0$ both networks are isolated and the transition is continuous. They found that the transition could change from discontinuous to continuous as the interdependency fraction is reduced below a critical fraction q_c, which is analogous to the critical point of the van der Waals phase diagram.

Later work studied the effect of targeted autonomization in which a fraction of nodes with the highest degrees in each network are set as autonomous [97, 98]. They obtained that the robustness of the GC is significantly improved compared to random autonomization because nodes with the highest degrees tend to maintain the GC connected. Moreover, Ref. [98] showed that for heterogeneous networks, such as SF networks, the system might have two transitions: one continuous and the other discontinuous, and three different characteristic GC sizes with thresholds that converge into a triple point, similar to the triple point in the phase diagram of water. This third phase emerges because as the initial failure of $1-p$ fraction of nodes increases, after the first transition the system does not completely collapse, but is composed of a core of hubs that were autonomized and maintains the network's functionality.

Modular networks and interdependent hierarchical networks: While many previous works on interdependent networks have been developed on networks without internal structure, real systems have a community structure at a mesoscopic scale such as the brain, social and financial networks and infrastructure across cities. Reference [99] introduced one of the first models of interdependent networks with communities in which bridge nodes (see Glossary) are attacked first. For the case of two interdependent networks and regardless of the number of communities, they observed that the system undergoes at most two-phase transitions. The first transition is due to the disconnection between modules or communities (see Glossary) and the system splits into several interdependent communities; whereas the second transition corresponds to the collapse of the communities caused by the dependency links. However, Ref. [100] showed that if communities are organized in a hierarchical structure with n levels, such as the infrastructure at the levels of a city, state and country, the system decays abruptly, at most, at n different values of the initial damage. Each collapse corresponds to the disconnection of a hierarchy level. This result indicates that at a mesoscopic scale, the effect of community structure on the robustness of the system saturates as the number of communities increases, but higher organization levels of the network structure make the system more fragile.

Internal rules of functionality: Several alternative internal rules to the GC were explored such as k-core percolation [59, 101, 102], functional finite clusters [103, 104], overload model [95] and redundant interdependencies [105]. For instance, in Ref. [103] the cascade of failures was studied for interdependent networks in which a node does not fail internally if it belongs to a cluster of size $s \geq s^*$, where s^* is a threshold. They found that depending on the value of s^*, the transition is continuous or discontinuous. For small values of s^*, the transition tends to be continuous because in the limit of $s^* = 1$ the system behaves as a single network. This shows that not only interdependent links are necessary to generate an abrupt collapse, but the rule of internal failure is crucial for a cascade of failure and abrupt collapse.

Stability: On a global scale, the mean size of the GC at the steady state is usually used as a measure of the stability of an interdependent network. However, Ref. [106] pointed out that, conversely, from a microscopic point of view, the probability that a node belongs to the GC at the end of the cascade, may be a good measure for the reliability of a node. For two interdependent networks, the stability decreases significantly if either of the two networks is homogeneous, and the most stable system corresponds to two heterogeneous networks. This phenomenon is due to the hubs which have a higher probability of belonging to the GC and serve as its anchors in a network [106], which reaffirms the role of the hubs of preserving the GC as it was shown in previous works [97, 98].

Finite systems: While many works have been developed in the thermodynamic limit, real networks have a finite size of the order of several thousand of nodes [107] and hence, the results obtained from these networks could be heavily affected by finite size effects. To solve this problem, Ref. [108] developed a new approach based on a non-backtracking matrix to study the robustness of real interdependent networks that do not necessarily have a treelike structure which is useful to compute the transition point. Another work pointed out the importance of taking into account the fluctuations in the size of the GC to characterize the robustness of finite interdependent networks [107]. For these systems, they observed that the mode [109] of the size of the GC (given by the value of the size of the GC with the highest probability among realizations) shows an abrupt behaviour, indicating that in finite networks this magnitude is a better estimator of the abrupt nature of cascades of failures in real systems.

Generalization of interdependence to dynamic systems: One of the main features of the original model of interdependent networks [55] that allows an exact theory with generating functions in the thermodynamic limit is that the state of the nodes are irreversibly changed. However, there are systems in nature or society in which nodes can restore a previous state, such as, a neuron that goes from inactive to active due to external stimulation. Besides, another simplification of the original model is that the interactions between networks correspond only to dependency relations, but it is known that interaction among systems, such as in ecology, could be trophic, mutualistic or parasitic [110, 111]. Therefore a theory is needed to describe these general relations and reversible dynamics. Recently, Ref. [93] developed a model exploring the effect on dynamics of different types of interactions between two or more networks. The versatility of the model allowed the authors to explore cooperation and competitive interactions. They found that these interactions generate rich dynamics, including chaotic regimes and hysteresis which are absent in interdependent networks. Another important simplification of the original model is that the cascade of failure is based on the topological structure, ignoring the flow or load between systems which has been recently incorporated in Refs. [94, 112]. Ref. [94] studied the cascade of failures between two partial interacting systems due to overloads. In this model, when a link fails in one network, a fraction of its load is distributed uniformly across the other links in the same network, and the rest is distributed uniformly among the links in the other network. They found that an increasing distributed load among the systems reduces the probability that only one of the systems collapse completely, and increases the probability that both systems can resist or collapse simultaneously. Finally, using a sandpile dynamics model, Ref. [112] studied how the number of interconnections between two networks affects the cascade. In this work, the authors demonstrated that there is an optimum number of interconnections which minimizes the probability of a large cascade in each network, and showed that adding these links is beneficial only in interconnected networks, and not in isolated networks.

In the following section, we review the literature on the effect of spatial embedding on the cascade of failures.

4. Spatial interdependent networks

Although many of the real networks, such as infrastructures are embedded in space, the interdependent network models described above did not include spatial embedding. In this section, we will discuss models which are embedded in space. The embedding implies that nodes actually have physical locations and thus the link length is constrained by the geometrical distance between linked nodes [113, 114]. Indeed, the spatial embedding significantly influences many aspects of a single network such as, for example, the different percolation thresholds for two-dimensional square lattices ($p_c \cong 0.59$) and random regular (RR) networks ($p_c = 1/3$) with the same degree ($k = 4$) as the square lattice. It is worth noting, that in most of the early research on spatial interdependent networks, square lattices were used as the model choice. This is reasonable since all two-dimensionally embedded networks with short length links, will be in the same universality class [115] and thus the overall properties and phenomenology will be similar for all embedded networks. Nevertheless, we also show that relaxing the lattice structure (see top of Fig. 3), leads to equivalent results.

Incorporating spatial embedding into interdependent networks has been found to significantly influence the vulnerability of the network. The first study of such systems was performed in Ref. [88] analysing two interdependent square lattices with a restricted length of dependency links, that is, a node A in one network can depend on a node B in the second network within a distance (between them) smaller than r (see Fig. 4). Intuitively, one can realize that if $r = 0$ the two networks are identical square lattices (perfectly overlapping) and thus the percolation properties would be the same as for a single lattice, that is, the percolation transition at $p_c = 0.59$ is continuous rather than abrupt. However, Ref. [88] found that for the case of random initial failures and when r is increased, there exists a critical length $r_c \approx 8$ above which the transition becomes abrupt due to the appearance of large cascades. They also find that at small $r < r_c$ the collapse transition is continuous, while for large $r > r_c$ it is the abrupt, but the effect of the maximal vulnerability of the networks at $r = r_c$ and metastability of the networks for $r > r_c$, when a hole of a sufficient size destroys the entire network, found in Ref. [88] is present only below six dimensions, suggesting that as in regular percolation, six is the upper critical dimension of this problem. Later works expand these studies to randomly connected graphs [116] in which the length of the dependency links is defined as a chemical distance, and to high-dimensional lattices [117]. Similar model [118, 119] with diluted lattices and dependency links of length $r = 0$ also produces a continuous transition as the model of Ref. [88] does for $r < r_c$.

Reference [86] considered two interdependent lattices in the limit $r \to \infty$, that is, unrestricted length of randomly chosen dependency links, yet varied the fraction q of interdependent nodes (with $1-q$ nodes being autonomous). Surprisingly, and in contrast to non-embedded random networks [96], it was found (both analytically and via simulations) that for any level of interdependence $q > 0$ the system undergoes an abrupt, first-order transition. This is in marked contrast with non-embedded networks where a critical dependency $q_c > 0$ is found.

Later work combined the studies of Refs [86, 88] by varying both q and r simultaneously [87]. There it was found that r_c increases as q decreases and that the nature of the cascades can be vastly different for r close to r_c as opposed to $r \gg r_c$. Namely for r near r_c a cascade front develops and spreads radially outward whereas for $r < r_c$ there is no spatial propagation front. Further work, considered $n > 2$ interdependent networks and found that for $n > 11$ even interdependent networks with $r = 1$ have an abrupt transition [120].

The above described studies focus on random failures in spatial interdependent networks, yet the concept of spatiality led to the recognition that it is possible that an entire network or NONs will fail in tandem due to a 'localized attack'. This localized attack or failure could be due to an earthquake, extreme storm event or other reason. The first study on localized attack [121] found that, in contrast to random attack where a finite fraction of the network must fail in order for the system to collapse, for a localized attack, a finite number of nodes, constituting a zero fraction of the system (when $N \to \infty$) can fail, causing cascade and lead to the total system collapse. It was shown that for certain values of $\langle k \rangle$ and r there exist a metastable state in which a localized attack with a radius above a critical value r_c^h propagates as a cascade of failures and yield to collapse of the entire system, while localized attacks below that size will not propagate. The value of r_c^h does not depend on the system size but only on the mean connectivity $\langle k \rangle$ in each network or layer and on the dependency link length r (see Fig. 4). In the $\langle k \rangle - r$ phase space, they also found a stable state in which there is always a GC for any localized attack size ($r_c^h = \infty$), and an unstable state where the entire system collapses without any localized attack (see Fig. 4). This concept of localized attack was later extended also for the case of non-spatial networks where it was shown to often lead to distinct results from random failures [92].

More recent work has exchanged the spatiality roles of the dependency and connectivity links in spatial interactions. Rather than having the connectivity links defined by a square lattice with nearest neighbour links of length 1, these new models, called ζ-models, have considered connectivity links with links drawn from an exponential distribution such that $P(l) \propto \exp(-l/\zeta)$, where ζ is the characteristic

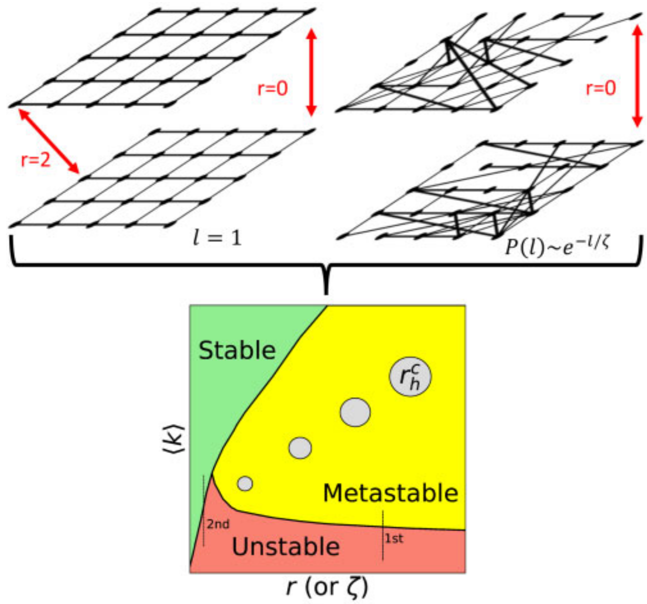

FIG. 4. Upper panel, left: schematic illustration of spatial interdependent networks (lattices) with dependency links that connect interdependent nodes within a distance r. Upper panel, right: schematic illustration of spatial interdependent networks with connectivity links of length l are taken from an exponential distribution, $P(l) \propto \exp(-l/\zeta)$ where ζ is the characteristic length of the connectivity links. Lower panel: schematic of the obtained phase diagram in the space $\langle k \rangle$ and r (or ζ) showing three phases: unstable, stable and metastable. In the metastable phase the critical radius r_h^c of the localized attack above which the system is destroyed, changes. The size of the circles indicates qualitatively that r_h^c increases with $\langle k \rangle$ and r (or ζ). Furthermore, the two vertical dashed lines represent the transitions for random percolation (reducing $\langle k \rangle$). For values of $r(\zeta)$ before the peak of the unstable red region, the transition is second order (continuous), whereas after the peak of the unstable red region the transition is first order (abrupt).

length of links (see Fig. 4). Two such sets of links can be constructed for the same set of nodes, and thus the dependency links have length zero, representing a spatial multiplex. This is more realistic since dependency on other infrastructures is more likely to be on nearby nodes; however, connectivity links for supply within a network can often involve larger distances. In Ref. [122], the authors developed this model and found that for ζ below some critical ζ_c (where p_c is maximal) the transition is continuous yet for larger ζ the transition is abrupt, in strong analogy with the results for r in Ref. [88] (see bottom of Fig. 4). Indeed, a more recent work considered localized attacks on the ζ-model and found that removing a finite number of nodes can also lead to total system collapse [123] (similar to Ref. [121]). These two studies have demonstrated a strong analogy between dependency links of length r with the connectivity of a spatial lattice and connectivity links from an exponential distribution with mean length ζ and dependency links of length zero. Both lead to a similar phase diagram, as seen in Fig. 4. The reason for this similarity is that each type of links can bring the failures to distances of r or ζ.

In Ref. [103], the authors used another internal rule of failure in which nodes survive if they belong to the GC or finite clusters with sizes greater than s^* because nodes in large finite components could still receive sufficient resources to continue functioning. Remarkably, the authors found a region of values of r in which the failure propagates as moving interfaces that belong to the universality class of a quenched Kardar–Parisi–Zhang equation [124].

5. Mitigation and recovery strategies

While the problem of cascade of failures has been extensively studied in the last decade, researchers from different areas have begun recently to develop strategies to mitigate the effects of the cascades and avoiding system collapse. These strategies have been focused not only on mitigation of the cascade but also on the recovery of the systems. For instance, for overload propagation, some strategies are based on systematically removing some nodes [125] or changing the coupling between systems [126]. On the other hand, for interdependent networks, several strategies consist of adding connectivity links through a healing process [127, 128], while others increase the functional GC as the failure propagates [129]. In this section, we will review some of the strategies where more details can be found in the cited scientific literature.

5.1 *Strategies to mitigate overload*

Motivated by the model presented in Ref. [5], researchers proposed and studied strategies to mitigate the cascade of failure due to overloads in single isolated networks [125]. These strategies consist of intentionally removing a fraction of nodes just after the initial failure, based on the smallest load, degree and closeness centrality. All these measures are almost equally effective in halting the cascade of failures because they are highly correlated. The author found that there exists an optimal value for the fraction of intentionally removed nodes for which the size of the GC at the end of the cascade is maximum.

Another proposed strategy [130] to mitigate the cascading failure due to overload was based on a spontaneous self-healing mechanism. In the overload cascade, all the nodes have the same load tolerance and a node fails when exceed its capacity. Then, it distributes its load among all of its functional neighbours. The model starts with the failure of the highest degree node in the network. Restoration begins at time t_r after the initial failure and consists in restoring a fraction of the failed nodes, giving them a new load tolerance higher than the initial one. The authors found that there exists an optimal restoration timing at which the size of the functional network is maximum.

In Ref. [126], the authors developed a model of two interacting networks, based on Ref. [94], where the interaction is produced by the load or coupling passing from one network to the other. In their model, part of the load of a failed line is sent to the other network through the coupling while the other part is redistributed within the first network. Even though the authors do not propose a strategy to avoid the cascade, they studied an optimal range for the coupling (i.e. the fraction of load in one network that is distributed to the other networks) of each network that maximizes the robustness of the system. They found that some values of coupling between the two networks increase the probability that both networks survive. An interesting consequence of this model is that it exhibits multiple discontinuous transitions, but in which the number of iterations to reach the steady state diverges only in the final breakdown of the network.

5.2 *Mitigation by healing nodes*

References [127, 128] proposed a healing model to mitigate the cascade of failures in embedded spatial networks. The strategy is applied at each time step of the cascading failure. The healing process consists of connecting with a probability ω, the nearest neighbours of each failed node among them (see Fig. 5a). The idea of this model is to replace the connections that were lost when the nodes failed. The authors found that the discontinuous transition can be turned into a continuous transition depending on the parameter ω. Below a threshold value ω_c, the nodes connect local nodes and both networks still preserve the structure of a lattice and the transition is abrupt. However, for $\omega \geq \omega_c$, the transition is continuous, the healing

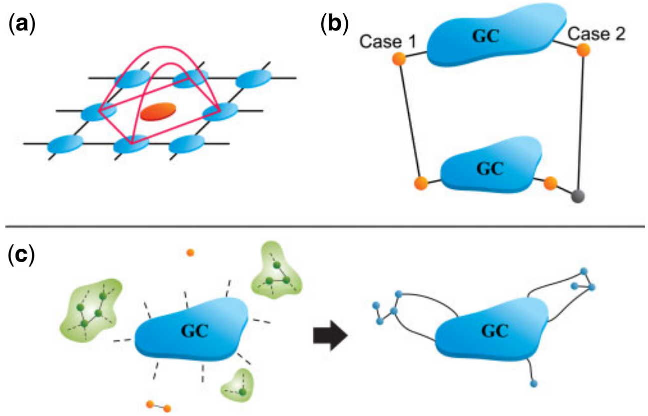

FIG. 5. Schematic of different strategies for mitigation of interdependent networks. (a) Healing strategy for interdependent lattices proposed in Refs. [127, 128]. The healing process consists of creating links (red) with a probability ω between the nearest neighbours of each failed node (red). (b) Schematic of a recovery strategy presented in Ref. [129]. The dysfunctional nodes at a distance one to their GC are in red, while the rest of the dysfunctional nodes at higher distances are in black. If a pair of dysfunctional interdependent nodes are at a unit distance from their GC, they are restored with probability γ (Case 1). However, if one of them is not at distance one from the GC, they cannot be restored (Case 2). (c) Schematic of a healing strategy for non-embedded interdependent networks presented in Ref. [131]. On the left, there is a snapshot of the GC and the cluster distribution in one layer during the cascade of failure: clusters that have not been saved by the strategy are in red, and those that will be reconnected to the GC are in green. Dashed links lead to dysfunctional nodes (not shown). On the right, some finite clusters have been reconnected to the GC.

promotes the formation of densely connected regions, connectivity links begin to join distant nodes, and the structure of the lattice is lost.

Another strategy that is similar to the healing process is the model proposed in Ref. [131] which consists in saving finite clusters before they fail (see Fig. 5c). During the cascading, each new finite cluster is attached to the GC with a finite probability. They found that as this probability increases the system can support a higher level of damage before the functional components collapse. While the transition is still discontinuous, the jump in the size of the GC at the transition point is reduced as the probability to attach clusters increases.

5.3 *Repair and recovery strategies in interdependent networks*

Until now, we have discussed strategies to mitigate the effects of the cascade of failures. However, a few models have been proposed with the purpose not only to mitigate cascades but also to recover part of the affected system. In Ref. [132], the authors generalized the model of recovery and failure in single networks developed in Ref. [47] to multiple networks. They found the minimal fraction of nodes needed to be repaired in each system to bring the entire system into the functional state. This set corresponds to the minimal Manhattan distance in the phase diagram that connects the starting point in which all networks are dysfunctional to the region where all of networks are functional. Using a mean field approach, for two interdependent systems they uncovered a hysteresis and a rich phase diagram with multiple triple points and four stable solutions following three scenarios: (i) both networks are in high activity, that is, functionality (ii) both networks are at low activity and (iii) only one network is functional while the other fails.

Recently, a repairing strategy has been proposed that not only mitigates the cascade of failures of the model in Ref. [55] but also can repair a failed system [129]. The rules for the internal and external failures are the same as in Ref. [55]. The strategy consists of repairing with a probability γ each pair of interdependent nodes that belongs to the perimeter of the GC in each network, where the perimeter is defined as the set of failed nodes at a chemical distance one from the GC (see Fig. 5b). The authors found that as γ increases, the system becomes more resilient to the cascade of failures. Moreover, the authors found that above a critical threshold of probability of repairing, the system is fully restored.

6. Conclusions

Cascading failures is one of the most important processes in complex networks which show how perturbations induce further failures that finally could lead to abrupt transitions at a global scale. The presence of dependency between different components is crucial for the propagation of a cascade. The models of interdependent complex networks discussed here are more vulnerable than single networks because fewer failures lead to system collapse. Moreover, spatial interdependent networks under localized attack can be even more vulnerable because of the space constraints. In recent years, studies in this field have increased dramatically, and new avenues of research continue to be developed, leading to a broader and deeper understanding of the origin and the effects of cascades of failures. For example, interacting networks may not only negatively affect each other [112, 133]. Often, additional networks are specially built to increase the reliability of the original networks. For example, SCADA is built to increase the reliability of the power grid. If the malfunction of SCADA elements never leads to malfunction of the power grid elements, its presence can only increase the robustness of the power grid. What makes interdependent networks more vulnerable is the particular property of interdependence: the nodes in one network cannot function without support from the nodes on which they depend. In fact, addition of various type of protection, such as local control stations [63], uninterrupted power supply for SCADA units, smart grid systems, increases the robustness of interdependent networks. For example, as pointed out in Ref. [64], the models of smart grid in which the failure of the control units of the communication network does not lead to the failure of a power station which they serve, the robustness increases with coupling strength between the power grid and communication network. However, if the failure of the communication node leads to the failure of the power node, the robustness decreases with the coupling strength in agreement with the predictions of topological models, for example [96]. In any case, analysis of real catastrophic blackouts demonstrates that failures of the control cyber-systems often plays the crucial role in their development [53, 134]. Further examples of very recent studies include understanding the stability of the GC [106], finite sizes effects [58, 108] and developing strategies to avoid an abrupt collapse of the system [128, 131].

Although several lines of research have provided an underlying framework, there are still many open questions that remain to be addressed. For instance, one of the significant problems is the lack of real data on multilayer networks that could be used to develop more realistic models for cascades of failures. Therefore, the construction of inference methods such as those suggested in Ref. [135], are necessary to advance our knowledge. Another problem is that most of the works in interdependent networks rely heavily on the network structure, but functionality can be based on the dynamics, such as in Refs. [93, 136]. This dynamic interdependency research is still in its infancy, and deeper studies should emerge. In this direction, it would be particularly significant, for example, to include interdependencies in the models proposed in Refs [42, 43], which would provide a more comprehensive elucidation of these processes. Finally, regarding strategies to contain and mitigate a cascade of failures, it would also be relevant to explore the optimal time and location to apply such strategies since resources are typically limited.

Funding

Defense Threat Reduction Agency (HDTRA1-14-1-0017); UNMdP and CONICET (PIP 00443/2014); and the National Science Foundation (PHY-1505000).

Acknowledgements

The authors thank Prof. H. Eugene Stanley and Prof. Hênio Henrique Aragão Rêgo for useful discussions.

Non-embedded and embedded networks in an Euclidean space

There exist real systems in which the topology and the process that develop on top of them are not constrained by the Euclidean distance, such as in the World Wide Web or Facebook. These systems are better modelled by non-embedded networks, and the simplest case is the Erdős–Rényi graph. One of the most remarkable properties of these systems is that they are small world, that is, the chemical distance or hop count between two nodes increases very slowly with the system size, as $\ln N$ or $\ln \ln N$. On the other hand, there are networks in which the topology is constrained by the space since nodes cannot connect to others that are spatially too far away, such as the road network between junctions or cities. The simplest model of an embedded network is a lattice. In these systems, the average distance increases faster than in non-embedded networks, as $N^{1/d}$ where d is the dimension of space of the embedded network. The small-world property in non-embedded networks implies that different processes on top of these networks such as a cascade of failures or the propagation of a rumor are much faster than in a Euclidean lattice.

Glossary

- Cascade of failures: Propagation of a failure through the system where a component that becomes dysfunctional lead to other components that depend on it (directly or indirectly) to also become dysfunctional.
- GC: A cluster with a macroscopic number of nodes.
- Finite cluster: A cluster with a microscopic number of nodes.
- Connectivity link: A link that connects two nodes from the same network.
- Dependency link: A link that connects a pair of nodes such that if one fails, then the other one also fails, even if still connected to the GC.
- Autonomous node: A node in an interdependent network that does not need any dependency link.
- Continuous phase transition: A phase transition such that at the critical threshold the order parameter (e.g. the magnetization in the Ising model or the size of the GC in percolation) approaches to zero continuously and many properties behave as power laws when approaching this threshold. If the derivative of the order parameter is discontinuous, then the transition is of second order.
- Discontinuous phase transition: A phase transition in which at a threshold value the order parameter is discontinuous and changes abruptly. It is usually called a ‘first-order transition’.
- Random failure, targeted and localized attack: Nodes can fail or be removed initially using different rules: at random (random failure), according to their degree or their centrality (targeted attack), or in nearby shells around a node (localized attack).
- Clustering: It is a measure of the density of triangles.
- Degree–degree correlation: Related to the probability that a node with degree k is connected to a node with a similar or different degree.

Glossary

- Modular network: A network composed of groups with a high internal density of connectivity links and a sparse density of connectivity links between these groups nearby (called modules or communities). The nodes that connect between two or more communities are called 'bridge nodes'. Modular networks can be found, for example, in metabolic systems, neural networks, social networks or infrastructures [137–140].
- Betweenness centrality: The fraction of shortest paths between all pair of nodes passing through a given node or a link [141].
- k-shell: It is obtained by removing, iteratively, all nodes with degree smaller than k, until all remaining nodes have degree k or larger [142].
- Hierarchical network: a modular network in which each module is formed by other modules, and so on. For example, countries could represent modules and, at the same time, each province, state or region in each country could also have modules or sub-modules.

Percolation in single networks

Percolation theory studies the statistical properties of clusters of nodes generated when links or nodes are removed from a network. Many physicists and mathematicians made contributions to this field; showing how percolation can describe different phenomena on Euclidean networks such as forest fires, the spread of diseases and electric conduction in disordered environments [143]. Currently, the theory of percolation has had a boom with the emergence of complex networks, since it allows researchers to describe and understand phenomena and processes such as the robustness of very heterogeneous networks against random failures and the difficulty of eliminating a virus from the Internet network [143–145]. One of the most studied percolation processes is random node or link percolation, in which a fraction $1 - p$ of nodes or links are removed randomly. This process exhibits a second-order transition at a critical value $p = p_c$ above which a GC exists, while below p_c the system contains only finite clusters. Other percolation processes that lead to cascades are k-core and bootstrap percolation [12–15, 142]. The former is used to study the deactivation process in which a fraction of nodes fail initially and then a node fails or becomes dysfunctional if it has a number of active neighbours below a chosen threshold. On the other hand, bootstrap percolation is an activation process, in which a fraction of nodes are initially activated, and then a node becomes functional if the number of active neighbours is above a threshold.

References

1. Newman, M. E. (2018) *Networks*. Oxford: Oxford University Press.
2. Barrat, A., Barthelemy, M. & Vespignani, A. (2008) *Dynamical Processes on Complex Networks*. Cambridge: Cambridge University Press.
3. Carreras, B. A., Newman, D. E. & Dobson, I. (2016) North American blackout time series statistics and implications for blackout risk. *IEEE Trans. Power Syst.*, **31**, 4406–4414.
4. Dobson, I. & Lu, L. (1992) Voltage collapse precipitated by the immediate change in stability when generator reactive power limits are encountered. *IEEE Trans. Circuits Syst. I*, **39**, 762–766.
5. Motter, A. E. & Lai, Y.-C. (2002) Cascade-based attacks on complex networks. *Phys. Rev. E*, **66**, 065102.
6. Chang, S. E. (2009) Infrastructure resilience to disasters. *The Bridge*, **39**, 36–41.

7. COMFORT, L. K. & HAASE, T. W. (2006) Communication, coherence, and collective action: the impact of Hurricane Katrina on communications infrastructure. *Public Works Manag. Policy*, **10**, 328–343.
8. LEAVITT, W. M. & KIEFER, J. J. (2006) Infrastructure interdependency and the creation of a normal disaster: the case of Hurricane Katrina and the city of New Orleans. *Public Works Manag. Policy*, **10**, 306–314.
9. KLOSIK, D. F., GRIMBS, A., BORNHOLDT, S. & HÜTT, M.-T. (2017) The interdependent network of gene regulation and metabolism is robust where it needs to be. *Nat. Commun.*, **8**, 534.
10. REIS, S. D., HU, Y., BABINO, A., ANDRADE JR, J. S., CANALS, S., SIGMAN, M. & MAKSE, H. A. (2014) Avoiding catastrophic failure in correlated networks of networks. *Nat. Phys.*, **10**, 762.
11. PARSHANI, R., BULDYREV, S. V. & HAVLIN, S. (2011) Critical effect of dependency groups on the function of networks. *Proc. Natl. Acad. Sci. USA*, **108**, 1007–1010.
12. BAXTER, G. J., DOROGOVTSEV, S. N., GOLTSEV, A. V. & MENDES, J. F. (2010) Bootstrap percolation on complex networks. *Phys. Rev. E*, **82**, 011103.
13. BAXTER, G. J., DOROGOVTSEV, S. N., GOLTSEV, A. V. & MENDES, J. F. F. (2011) Heterogeneous k-core versus bootstrap percolation on complex networks. *Phys. Rev. E*, **83**, 051134.
14. DI MURO, M. A., VALDEZ, L. D., BULDYREV, S. V., STANLEY, H. E. & BRAUNSTEIN, L. A. (2019) Insights into bootstrap percolation: its equivalence with k-core percolation and the giant component. *Phys. Rev. E*, **99**, 022311.
15. DOROGOVTSEV, S. N., GOLTSEV, A. V. & MENDES, J. F. F. (2006) K-core organization of complex networks. *Phys. Rev. Lett.*, **96**, 040601.
16. BRESKIN, I., SORIANO, J., MOSES, E. & TLUSTY, T. (2006) Percolation in living neural networks. *Phys. Rev. Lett.*, **97**, 188102.
17. MONCEAU, P., MÉTENS, S., FARDET, T., RENAULT, R. & BOTTANI, S. (2018) Neuronal devices: understanding neuronal cultures through percolation helps prepare for the next step. *Eur. Phys. J. Spec. Top.*, **227**, 1015–1028.
18. SORIANO, J., MARTÍNEZ, M. R., TLUSTY, T. & MOSES, E. (2008) Development of input connections in neural cultures. *Proc. Natl. Acad. Sci. USA*, **105**, 13758–13763.
19. MØNSTED, B., SAPIEŻYŃSKI, P., FERRARA, E. & LEHMANN, S. (2017) Evidence of complex contagion of information in social media: an experiment using Twitter bots. *PLoS One*, **12**, e0184148.
20. ROMERO, D. M., MEEDER, B. & KLEINBERG, J. (2011) Differences in the mechanics of information diffusion across topics: idioms, political hashtags, and complex contagion on twitter. *Proceedings of the 20th International Conference on World Wide Web*. ACM, pp. 695–704.
21. ZHAO, J., LI, D., SANHEDRAI, H., COHEN, R. & HAVLIN, S. (2016) Spatio-temporal propagation of cascading overload failures in spatially embedded networks. *Nat. Commun.*, **7**, 10094.
22. ZHANG, L., ZENG, G., LI, D., HUANG, H.-J., STANLEY, H. E. & HAVLIN, S. (2019) Scale-free resilience of real traffic jams. *Proc. Natl. Acad. Sci. USA*, 201814982.
23. KORNBLUTH, Y., BARACH, G., TUCHMAN, Y., KADISH, B., CWILICH, G. & BULDYREV, S. V. (2018) Network overload due to massive attacks. *Phys. Rev. E*, **97**, 052309.
24. CARRERAS, B. A., LYNCH, V. E., DOBSON, I. & NEWMAN, D. E. (2002) Critical points and transitions in an electric power transmission model for cascading failure blackouts. *Chaos*, **12**, 985–994.
25. CARRERAS, B. A., LYNCH, V. E., DOBSON, I. & NEWMAN, D. E. (2004) Complex dynamics of blackouts in power transmission systems. *Chaos*, **14**, 643–652.
26. REN, H., DOBSON, I. & CARRERAS, B. A. (2008) Long-term effect of the n-1 criterion on cascading line outages in an evolving power transmission grid. *IEEE Trans. Power Syst.*, **23**, 1217–1225.
27. CARRERAS, B. A., NEWMAN, D. E., DOBSON, I. & POOLE, A. B. (2000) Initial evidence for self-organized criticality in electric power blackouts. *33rd Hawaii International Conference on System Sciences, Maui*. IEEE Computer Society Press.
28. CARRERAS, B. A., NEWMAN, D. E., DOBSON, I. & POOLE, A. B. (2001) Evidence for self-organized criticality in electric power system blackouts. *34th Hawaii International Conference on System Sciences, Maui, Hawaii*. IEEE Computer Society Press.
29. CARRERAS, B. A., NEWMAN, D. E., DOBSON, I. & POOLE, A. B. (2004) Evidence for self-organized criticality in a time series of electric power system blackouts. *IEEE Trans. Circuits Syst. I*, **51**, 1733–1740.

30. Dobson, I. (2012) Estimating the propagation and extent of cascading line outages from utility data with a branching process. *IEEE Trans. Power Syst.*, **27**, 2146–2155.
31. Dobson, I., Carreras, B. A., Lynch, V. E. & Newman, D. E. (2007) Complex systems analysis of series of blackouts: cascading failure, critical points, and self-organization. *Chaos*, **17**, 026103.
32. Kim, J., Wierzbicki, K. R., Dobson, I. & Hardiman, R. C. (2012) Estimating propagation and distribution of load shed in simulations of cascading blackouts. *IEEE Syst. J.*, **6**, 548–557.
33. Newman, D. E., Carreras, B. A., Lynch, V. E. & Dobson, I. (2011) Exploring complex systems aspects of blackout risk and mitigation. *IEEE Trans. Reliab.*, **60**, 134–143.
34. Qi, J., Dobson, I. & Mei, S. (2013) Towards estimating the statistics of simulated cascades of outages with branching processes. *IEEE Trans. Power Syst.*, **28**, 3410–3419.
35. Bak, P. & Sneppen, K. (1993) Punctuated equilibrium and criticality in a simple model of evolution. *Phys. Rev. Lett.*, **71**, 4083.
36. Maslov, S. (1995) Time directed avalanches in invasion models. *Phys. Rev. Lett.*, **74**, 562.
37. Maslov, S. & Zhang, Y.-C. (1995) Exactly solved model of self-organized criticality. *Phys. Rev. Lett.*, **75**, 1550.
38. Paczuski, M., Maslov, S. & Bak, P. (1996) Avalanche dynamics in evolution, growth, and depinning models. *Phys. Rev. E*, **53**, 414.
39. Pahwa, S., Scoglio, C. & Scala, A. (2014) Abruptness of cascade failures in power grids. *Sci. Rep.*, **4**, 3694.
40. Spiewak, R., Soltan, S., Forman, Y., Buldyrev, S. & Zussman, G. (2018) A study of cascading failures in real and synthetic power grid topologies. *Netw. Sci.*, **6**, 448–468.
41. Hines, P., Cotilla-Sanchez, E. & Blumsack, S. (2010) Do topological models provide good information about electricity infrastructure vulnerability? *Chaos*, **20**, 033122.
42. Schäfer, B., Witthaut, D., Timme, M. & Latora, V. (2018) Dynamically induced cascading failures in power grids. *Nat. Commun.*, **9**, 1975.
43. Yang, Y. & Motter, A. E. (2017) Cascading failures as continuous phase-space transitions. *Phys. Rev. Lett.*, **119**, 248302.
44. Yang, Y., Nishikawa, T. & Motter, A. E. (2017) Vulnerability and cosusceptibility determine the size of network cascades. *Phys. Rev. Lett.*, **118**, 048301.
45. Wang, Z., Zhou, D. & Hu, Y. (2018) Group percolation in interdependent networks. *Phys. Rev. E*, **97**, 032306.
46. Yang, Y., Nishikawa, T. & Motter, A. E. (2017) Small vulnerable sets determine large network cascades in power grids. *Science*, **358**, eaan3184.
47. Majdandzic, A., Podobnik, B., Buldyrev, S. V., Kenett, D. Y., Havlin, S. & Stanley, H. E. (2014) Spontaneous recovery in dynamical networks. *Nat. Phys.*, **10**, 34.
48. Böttcher, L., Luković, M., Nagler, J., Havlin, S. & Herrmann, H. J. (2017) Failure and recovery in dynamical networks. *Sci. Rep.*, **7**, 41729.
49. Valdez, L. D., Di Muro, M. A. & Braunstein, L. A. (2016) Failure-recovery model with competition between failures in complex networks: a dynamical approach. *J. Stat. Mech. Theory Exp.*, **2016**, 093402.
50. Goltsev, A. V., De Abreu, F. V., Dorogovtsev, S. N. & Mendes, J. F. F. (2010) Stochastic cellular automata model of neural networks. *Phys. Rev. E*, **81**, 061921.
51. Rinaldi, S. M., Peerenboom, J. P. & Kelly, T. K. (2001) Identifying, understanding, and analyzing critical infrastructure interdependencies. *IEEE Control Syst.*, **21**, 11–25.
52. Vespignani, A. (2010) Complex networks: the fragility of interdependency. *Nature*, **464**, 984.
53. Rosato, V., Issacharoff, L., Tiriticco, F., Meloni, S., Porcellinis, S. & Setola, R. (2008) Modelling interdependent infrastructures using interacting dynamical models. *Int. J. Crit. Infrastruct.*, **4**, 63–79.
54. Bianconi, G. (2018) *Multilayer Networks: Structure and Function*. Oxford: Oxford University Press.
55. Buldyrev, S. V., Parshani, R., Paul, G., Stanley, H. E. & Havlin, S. (2010) Catastrophic cascade of failures in interdependent networks. *Nature*, **464**, 1025.
56. D'Agostino, G. & Scala, A. (2014) *Networks of Networks: The Last Frontier of Complexity*, vol. 340. Berlin: Springer.

57. Baxter, G. J., Dorogovtsev, S. N., Goltsev, A. V. & Mendes, J. F. F. (2012) Avalanche collapse of interdependent networks. *Phys. Rev. Lett.*, **109**, 248701.
58. Zhou, D., Bashan, A., Cohen, R., Berezin, Y., Shnerb, N. & Havlin, S. (2014) Simultaneous first- and second-order percolation transitions in interdependent networks. *Phys. Rev. E*, **90**, 012803.
59. Di Muro, M. A., Valdez, L. D., Rêgo, H. H. A., Buldyrev, S. V., Stanley, H. E. & Braunstein, L. A. (2017) Cascading failures in interdependent networks with multiple supply-demand links and functionality thresholds. *Sci. Rep.*, **7**, 15059.
60. Serrano, M. Á., Buzna, L. & Boguná, M. (2015) Escaping the avalanche collapse in self-similar multiplexes. *New J. Phys.*, **17**, 053033.
61. Son, S.-W., Bizhani, G., Christensen, C., Grassberger, P. & Paczuski, M. (2012) Percolation theory on interdependent networks based on epidemic spreading. *Euro Phys. Lett.*, **97**, 16006.
62. Stauffer, D. & Aharony, A. (1994) *Introduction to Percolation Theory: Revised second edition*. London: Francis & Taylor.
63. Rezai, A., Keshavarzi, P. & Moravej, Z. (2017) Key management issue in SCADA networks: a review. *Eng. Sci. Technol.*, **20**, 354–363.
64. Korkali, M., Veneman, J. G., Tivnan, B. F., Bagrow, J. P. & Hines, P. D. (2017) Reducing cascading failure risk by increasing infrastructure network interdependence. *Sci. Rep.*, **7**, 44499.
65. Buldyrev, S. V., Shere, N. W. & Cwilich, G. A. (2011) Interdependent networks with identical degrees of mutually dependent nodes. *Phys. Rev. E*, **83**, 016112.
66. Min, B., Do Yi, S., Lee, K.-M. & Goh, K.-I. (2014) Network robustness of multiplex networks with interlayer degree correlations. *Phys. Rev. E*, **89**, 042811.
67. Valdez, L. D., Macri, P. A., Stanley, H. E. & Braunstein, L. A. (2013) Triple point in correlated interdependent networks. *Phys. Rev. E*, **88**, 050803.
68. Watanabe, S. & Kabashima, Y. (2014) Cavity-based robustness analysis of interdependent networks: influences of intranetwork and internetwork degree-degree correlations. *Phys. Rev. E*, **89**, 012808.
69. Zhou, D., Stanley, H. E., DAgostino, G. & Scala, A. (2012) Assortativity decreases the robustness of interdependent networks. *Phys. Rev. E*, **86**, 066103.
70. Baxter, G. J., Bianconi, G., da Costa, R. A., Dorogovtsev, S. N. & Mendes, J. F. F. (2016) Correlated edge overlaps in multiplex networks. *Phys. Rev. E*, **94**, 012303.
71. Cellai, D., Dorogovtsev, S. N. & Bianconi, G. (2016) Message passing theory for percolation models on multiplex networks with link overlap. *Phys. Rev. E*, **94**, 032301.
72. Cellai, D., López, E., Zhou, J., Gleeson, J. P. & Bianconi, G. (2013) Percolation in multiplex networks with overlap. *Phys. Rev. E*, **88**, 052811.
73. Hu, Y., Zhou, D., Zhang, R., Han, Z., Rozenblat, C. & Havlin, S. (2013) Percolation of interdependent networks with intersimilarity. *Phys. Rev. E*, **88**, 052805.
74. Min, B., Lee, S., Lee, K.-M. & Goh, K.-I. (2015) Link overlap, viability, and mutual percolation in multiplex networks. *Chaos Solitons Fractals*, **72**, 49–58.
75. Huang, X., Shao, S., Wang, H., Buldyrev, S. V., Stanley, H. E. & Havlin, S. (2013) The robustness of interdependent clustered networks. *Euro Phys. Lett.*, **101**, 18002.
76. Shao, S., Huang, X., Stanley, H. E. & Havlin, S. (2014) Robustness of a partially interdependent network formed of clustered networks. *Phys. Rev. E*, **89**, 032812.
77. Tian, L., Huang, Y., Dong, G., Du, R. & Shi, L. (2014) Robustness of interdependent and interconnected clustered networks. *Physica A*, **412**, 120–126.
78. Liu, X., Pan, L., Stanley, H. E. & Gao, J. (2019) Multiple phase transitions in networks of directed networks. *Phys. Rev. E*, **99**, 012312.
79. Liu, X., Stanley, H. E. & Gao, J. (2016) Breakdown of interdependent directed networks. *Proc. Natl. Acad. Sci. USA*, **113**, 1138–1143.
80. Bianconi, G. & Dorogovtsev, S. N. (2014) Multiple percolation transitions in a configuration model of a network of networks. *Phys. Rev. E*, **89**, 062814.

81. BIANCONI, G., DOROGOVTSEV, S. N. & MENDES, J. F. (2015) Mutually connected component of networks of networks with replica nodes. *Phys. Rev. E*, **91**, 012804.
82. GAO, J., BULDYREV, S. V., HAVLIN, S. & STANLEY, H. E. (2011) Robustness of a network of networks. *Phys. Rev. Lett.*, **107**, 195701.
83. GAO, J., BULDYREV, S. V., STANLEY, H. E. & HAVLIN, S. (2012) Networks formed from interdependent networks. *Nat. Phys.*, **8**, 40.
84. GAO, J., BULDYREV, S. V., STANLEY, H. E., XU, X. & HAVLIN, S. (2013) Percolation of a general network of networks. *Phys. Rev. E*, **88**, 062816.
85. SHAO, J., BULDYREV, S. V., HAVLIN, S. & STANLEY, H. E. (2011) Cascade of failures in coupled network systems with multiple support-dependence relations. *Phys. Rev. E*, **83**, 036116.
86. BASHAN, A., BEREZIN, Y., BULDYREV, S. V. & HAVLIN, S. (2013) The extreme vulnerability of interdependent spatially embedded networks. *Nat. Phys.*, **9**, 667.
87. DANZIGER, M. M., BASHAN, A., BEREZIN, Y. & HAVLIN, S. (2014) Percolation and cascade dynamics of spatial networks with partial dependency. *J. Compl. Netw.*, **2**, 460–474.
88. LI, W., BASHAN, A., BULDYREV, S. V., STANLEY, H. E. & HAVLIN, S. (2012) Cascading failures in interdependent lattice networks: the critical role of the length of dependency links. *Phys. Rev. Lett.*, **108**, 228702.
89. KLEINEBERG, K.-K., BUZNA, L., PAPADOPOULOS, F., BOGUÑÁ, M. & SERRANO, M. A. (2017) Geometric correlations mitigate the extreme vulnerability of multiplex networks against targeted attacks. *Phys. Rev. Lett.*, **118**, 218301.
90. BAXTER, G. J., TIMÁR, G. & MENDES, J. F. F. (2018) Targeted damage to interdependent networks. *Phys. Rev. E*, **98**, 032307.
91. HUANG, X., GAO, J., BULDYREV, S. V., HAVLIN, S. & STANLEY, H. E. (2011) Robustness of interdependent networks under targeted attack. *Phys. Rev. E*, **83**, 065101.
92. YUAN, X., SHAO, S., STANLEY, H. E. & HAVLIN, S. (2015) How breadth of degree distribution influences network robustness: comparing localized and random attacks. *Phys. Rev. E*, **92**, 032122.
93. DANZIGER, M. M., BONAMASSA, I., BOCCALETTI, S. & HAVLIN, S. (2019) Dynamic interdependence and competition in multilayer networks. *Nat. Phys.*, **15**, 178–185.
94. SCALA, A., LUCENTINI, P. G. D. S., CALDARELLI, G. & DAGOSTINO, G. (2016) Cascades in interdependent flow networks. *Physica D*, **323**, 35–39.
95. ZHANG, Y. & YAGAN, O. (2018) Modeling and analysis of cascading failures in interdependent cyber-physical systems. *2018 IEEE Conference on Decision and Control (CDC)*. IEEE, pp. 4731–4738.
96. PARSHANI, R., BULDYREV, S. V. & HAVLIN, S. (2010) Interdependent networks: reducing the coupling strength leads to a change from a first to second order percolation transition. *Phys. Rev. Lett.*, **105**, 048701.
97. SCHNEIDER, C. M., YAZDANI, N., ARAÚJO, N. A. M., HAVLIN, S. & HERRMANN, H. J. (2013) Towards designing robust coupled networks. *Sci. Rep.*, **3**, 1969.
98. VALDEZ, L. D., MACRI, P. A. & BRAUNSTEIN, L. A. (2014) A triple point induced by targeted autonomization on interdependent scale-free networks. *J. Phys. A*, **47**, 055002.
99. SHEKHTMAN, L. M., SHAI, S. & HAVLIN, S. (2015) Resilience of networks formed of interdependent modular networks. *New J. Phys.*, **17**, 123007.
100. SHEKHTMAN, L. M. & HAVLIN, S. (2018) Percolation of hierarchical networks and networks of networks. *Phys. Rev. E*, **98**, 052305.
101. AZIMI-TAFRESHI, N., GÓMEZ-GARDENES, J. & DOROGOVTSEV, S. N. (2014) k- Core percolation on multiplex networks. *Phys. Rev. E*, **90**, 032816.
102. PANDURANGA, N. K., GAO, J., YUAN, X., STANLEY, H. E. & HAVLIN, S. (2017) Generalized model for k-core percolation and interdependent networks. *Phys. Rev. E*, **96**, 032317.
103. DI MURO, M. A., BULDYREV, S. V., STANLEY, H. E. & BRAUNSTEIN, L. A. (2016) Cascading failures in interdependent networks with finite functional components. *Phys. Rev. E*, **94**, 042304.
104. YUAN, X., HU, Y., STANLEY, H. E. & HAVLIN, S. (2017) Eradicating catastrophic collapse in interdependent networks via reinforced nodes. *Proc. Natl. Acad. Sci. USA*, **114**, 3311–3315.

105. Radicchi, F. & Bianconi, G. (2017) Redundant interdependencies boost the robustness of multiplex networks. *Phys. Rev. X*, **7**, 011013.
106. Kitsak, M., Ganin, A. A., Eisenberg, D. A., Krapivsky, P. L., Krioukov, D., Alderson, D. L. & Linkov, I. (2018) Stability of a giant connected component in a complex network. *Phys. Rev. E*, **97**, 012309.
107. Coghi, F., Radicchi, F. & Bianconi, G. (2018) Controlling the uncertain response of real multiplex networks to random damage. *Phys. Rev. E*, **98**, 062317.
108. Radicchi, F. (2015) Percolation in real interdependent networks. *Nat. Phys.*, **11**, 597.
109. Manikandan, S. (2011) Measures of central tendency: median and mode. *J. Pharmacol. Pharmacother.*, **2**, 214.
110. Pilosof, S., Porter, M. A., Pascual, M. & Kéfi, S. (2017) The multilayer nature of ecological networks. *Nature Ecol. & Evol.*, **1**, 0101.
111. Pocock, M. J., Evans, D. M. & Memmott, J. (2012) The robustness and restoration of a network of ecological networks. *Science*, **335**, 973–977.
112. Brummitt, C. D., DSouza, R. M. & Leicht, E. A. (2012) Suppressing cascades of load in interdependent networks. *Proc. Natl. Acad. Sci. USA*, **109**, E680–E689.
113. Barthélemy, M. (2011) Spatial networks. *Phys. Rep.*, **499**, 1–101.
114. McAndrew, T. C., Danforth, C. M. & Bagrow, J. P. (2015) Robustness of spatial micronetworks. *Phys. Rev. E*, **91**, 042813.
115. Daqing, L., Kosmidis, K., Bunde, A. & Havlin, S. (2011) Dimension of spatially embedded networks. *Nat. Phys.*, **7**, 481.
116. Kornbluth, Y., Lowinger, S., Cwilich, G. & Buldyrev, S. V. (2014) Cascading failures in networks with proximate dependent nodes. *Phys. Rev. E*, **89**, 032808.
117. Lowinger, S., Cwilich, G. A. & Buldyrev, S. V. (2016) Interdependent lattice networks in high dimensions. *Phys. Rev. E*, **94**, 052306.
118. Berezin, Y., Bashan, A. & Havlin, S. (2013) Comment on percolation transitions are not always sharpened by making networks interdependent. *Phys. Rev. Lett.*, **111**, 189601.
119. Son, S.-W., Grassberger, P. & Paczuski, M. (2011) Percolation transitions are not always sharpened by making networks interdependent. *Phys. Rev. Lett.*, **107**, 195702.
120. Shekhtman, L. M., Berezin, Y., Danziger, M. M. & Havlin, S. (2014) Robustness of a network formed of spatially embedded networks. *Phys. Rev. E*, **90**, 012809.
121. Berezin, Y., Bashan, A., Danziger, M. M., Li, D. & Havlin, S. (2015) Localized attacks on spatially embedded networks with dependencies. *Sci. Rep.*, **5**, 8934.
122. Danziger, M. M., Shekhtman, L. M., Berezin, Y. & Havlin, S. (2016) The effect of spatiality on multiplex networks. *Euro Phys. Lett.*, **115**, 36002.
123. Vaknin, D., Danziger, M. M. & Havlin, S. (2017) Spreading of localized attacks in spatial multiplex networks. *New J. Phys.*, **19**, 073037.
124. Kardar, M., Parisi, G. & Zhang, Y.-C. (1986) Dynamic scaling of growing interfaces. *Phys. Rev. Lett.*, **56**, 889.
125. Motter, A. E. (2004) Cascade control and defense in complex networks. *Phys. Rev. Lett.*, **93**, 098701.
126. Zhang, Y., Arenas, A. & Yağan, O. (2018) Cascading failures in interdependent systems under a flow redistribution model. *Phys. Rev. E*, **97**, 022307.
127. Stippinger, M. & Kertész, J. (2014) Enhancing resilience of interdependent networks by healing. *Physica A*, **416**, 481–487.
128. Stippinger, M. & Kertész, J. (2018) Universality and scaling laws in the cascading failure model with healing. *Phys. Rev. E*, **98**, 042303.
129. Di Muro, M. A., La Rocca, C. E., Stanley, H. E., Havlin, S. & Braunstein, L. A. (2016) Recovery of interdependent networks. *Sci. Rep.*, **6**, 22834.
130. Liu, C., Li, D., Fu, B., Yang, S., Wang, Y. & Lu, G. (2014) Modeling of self-healing against cascading overload failures in complex networks. *Euro Phys. Lett.*, **107**, 68003.

131. La Rocca, C. E., Stanley, H. E. & Braunstein, L. A. (2018) Strategy for stopping failure cascades in interdependent networks. *Physica A*, **508**, 577.
132. Majdandzic, A., Braunstein, L. A., Curme, C., Vodenska, I., Levy-Carciente, S., Stanley, H. E. & Havlin, S. (2016) Multiple tipping points and optimal repairing in interacting networks. *Nat. Commun.*, **7**, 10850.
133. Leicht, E. A. & D'Souza, R. M. (2009) Percolation on interacting networks. ArXiv:0907.0894.
134. New York Independent System Operator (2004) Interim report on the August 14, 2003 blackout.
135. Lacasa, L., Mariño, I. P., Miguez, J., Nicosia, V., Roldán, É., Lisica, A., Grill, S. W. & Gómez-Gardeñes, J. (2018) Multiplex decomposition of non-Markovian dynamics and the hidden layer reconstruction problem. *Phys. Rev. X*, **8**, 031038.
136. Danziger, M. M., Bashan, A. & Havlin, S. (2015) Interdependent resistor networks with process-based dependency. *New J. Phys.*, **17**, 043046.
137. Eriksen, K. A., Simonsen, I., Maslov, S. & Sneppen, K. (2003) Modularity and extreme edges of the Internet. *Phys. Rev. Lett.*, **90**, 148701.
138. González, M. C., Herrmann, H. J., Kertész, J. & Vicsek, T. (2007) Community structure and ethnic preferences in school friendship networks. *Physica A*, **379**, 307–316.
139. Happel, B. L. M. & Murre, J. M. J. (1994) Design and evolution of modular neural network architectures. *Neural Netw.*, **7**, 985–1004.
140. Ravasz, E., Somera, A. L., Mongru, D. A., Oltvai, Z. N. & Barabási, A.-L. (2002) Hierarchical organization of modularity in metabolic networks. *Science*, **297**, 1551–1555.
141. Newman, M. E. J. (2010) *Networks: An Introduction*. Oxford: Oxford University Press.
142. Carmi, S., Havlin, S., Kirkpatrick, S., Shavitt, Y. & Shir, E. (2007) A model of Internet topology using k-shell decomposition. *Proc. Natl. Acad. Sci. USA*, **104**, 11150–11154.
143. Araújo, N., Grassberger, P., Kahng, B., Schrenk, K. & Ziff, R. (2014) Recent advances and open challenges in percolation. *Eur. Phys. J. Spec. Top.*, **223**, 2307–2321.
144. Boccaletti, S., Latora, V., Moreno, Y., Chavez, M. & Hwang, D.-U. (2006) Complex networks: structure and dynamics. *Phys. Rep.*, **424**, 175–308.
145. Cohen, R. & Havlin, S. (2010) *Complex Networks: Structure, Robustness and Function*. Cambridge: Cambridge University Press.