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Scaling behaviour in the growth of companies

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A successful theory of corporate growth should include both the external and internal factors that affect the growth of a company[1–18]. Whereas traditional models emphasize production-related influences such as investment in physical capital and in research and development[18], recent models[10–20] recognize the equal importance of organizational infrastructure. Unfortunately, no exhaustive empirical account of the growth of companies exists by which these models can be tested. Here we present a broad, phenomenological picture of the dependence of growth on company size, derived from data for all publicly traded US manufacturing companies between 1975 and 1991. We find that, for firms with similar sales, the distribution of annual (logarithmic) growth rates has an exponential form; the spread in the distribution of rates decreases with increasing sales as a power law over seven orders of magnitude. A model wherein the probability of a company's growth depends on its past as well as present sales accounts for the former observation. As the latter observation applies to companies that manufacture products of all kinds, organizational structures common to all firms might well be stronger determinants of growth than production-related factors, which differ for companies producing different goods.

The simplest model for corporate growth was proposed by Gibrat[1]. Its basic assumptions are that the rate of company growth is (1) independent of company size (law of proportionate effect), and (2) uncorrelated in time. These assumptions can be formalized by the following random multiplicative process: $S_{t+\Delta t} = S_t(1 + \varepsilon_t)$, where $S_{t+\Delta t}$ and S_t are the sales of the company at time $t + \Delta t$ and t respectively, and ε_t is an uncorrelated random number with mean close to zero and standard deviation much smaller than one. Hence $\log S_t$ follows a simple random walk so that firm sizes are log–normally distributed. Also, for sufficiently large time intervals $T \gg \Delta t$, the growth rates S_{t+T}/S_t are log–normally distributed.

Although it is known that Gibrat's assumptions are rejected empirically, many theoretical and empirical analyses still use the Gibrat model as a benchmark, for lack of a better alternative[21–26]. To achieve a more realistic characterization of company dynamics, we analyse the statistical properties of the growth rates.

We studied all US manufacturing publicly traded companies within the years 1975–91. The data were taken from the Compustat database and all values for sales have been adjusted to 1987 dollars by the GNP price deflator. (Compustat contains financial information that firms traded on United States securities exchanges must file with the US Securities and Exchange Commission.) We define a firm's annual growth rate as $R \equiv S_1/S_0$, where S_0 and S_1 are its sales in two consecutive years.

It is customary to study company growth on logarithmic scales, so we define $r \equiv \ln(S_1/S_0)$ and $s_0 \equiv \ln S_0$ and calculate the conditional distribution $p(r \mid s_0)$ of growth rates r with a given initial sales value s_0.

The distribution $p(r \mid s_0)$ of the growth rates from 1990 to 1991 is shown in Fig. 1*a* for two different values of initial sales. Remarkably, both curves display a simple 'tent-shaped' form. The distribution is not gaussian—as expected from the Gibrat model—but rather is exponential,

$$p(r \mid s_0) = \frac{1}{\sqrt{2}\sigma(s_0)} \exp\left(-\frac{\sqrt{2}|r - \bar{r}(s_0)|}{\sigma(s_0)}\right) \quad (1)$$

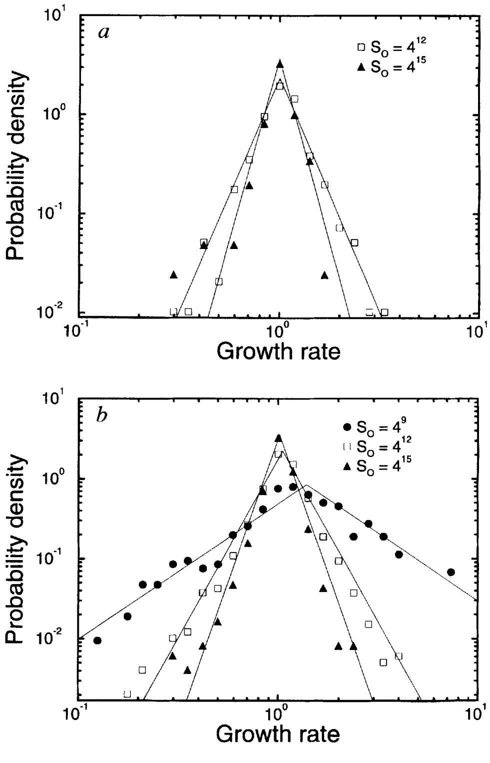

FIG. 1 *a*, Probability density $p(r \mid s_0)$ of the growth rate $r \equiv \ln(S_1/S_0)$ from year 1990 to 1991 for all publicly traded US manufacturing firms in the 1994 Compustat database with standard industrial classification index of 2000–3999. We examine 1991 because between 1992 and 1994 there are several companies with zero sales that either have gone out of business or are 'new technology' companies (developing new products). We show the data for two different bins of initial sales (with sizes increasing by powers of 4): $4^{11.5} < S_0 < 4^{12.5}$ (squares) and $4^{14.5} < S_0 < 4^{15.5}$ (triangles). Within each sales bin, each firm has a different value of R, so the abscissa value is obtained by binning these R values. The solid lines are fits to equation (1) (in the text) using the mean $\bar{r}(s_0)$ and standard deviation $\sigma(s_0)$ calculated from the data. *b*, Probability density $p(r \mid s_0)$ of the annual growth rate, for three different bins of initial sales: $4^{8.5} < S_0 < 4^{9.5}$ (circles), $4^{11.5} < S_0 < 4^{12.5}$ (squares) and $4^{14.5} < S_0 < 4^{15.5}$ (triangles). The data were averaged over all 16 one-year periods between 1975 and 1991. The solid lines are fits to equation (1) using the mean $\bar{r}(s_0)$ and standard deviation $\sigma(s_0)$ calculated from all data.

The straight lines shown in Fig. 1*a* are calculated from the average growth rate $\bar{r}(s_0)$ and the standard deviation $\sigma(s_0)$ obtained by fitting the data set to equation (1).

We also find that the data for each of the 16 annual intervals from the period 1975–91 fit well to equation (1), with only small variations in the parameters $\bar{r}(s_0)$ and $\sigma(s_0)$. To improve the statistics, we therefore calculate the new distribution by averaging all the data from the 16 annual intervals in the database. As shown in Fig. 1*b*, the data now scatter much less and the shape is well described by equation (1). For this reason, we have also included in the figure data for 'volatile' cases, corresponding to sales of only about 2.6×10^5 dollars.

As is apparent from Fig. 1*b*, $\sigma(s_0)$ decreases with increasing s_0. We find $\sigma(s_0)$ is well approximated over more than seven orders of magnitude—from sales of less than 10^4 dollars up to sales of more than 10^{11} dollars—by the law

$$\sigma(s_0) = a\exp(-\beta s_0) = aS_0^{-\beta} \quad (2)$$

where $a \approx 6.66$ and $\beta = 0.15 \pm 0.03$ (Fig. 2).

We performed a parallel analysis for the number of employees, and the corresponding standard deviation is shown in Fig. 2. The data are linear over roughly five orders of magnitude, from firms with only 10 employees to firms with almost 10^6 employees. The slope $\beta = 0.16 \pm 0.03$ is the same, within error bars, as that found for sales.

We find that equations (1) and (2) accurately describe three additional indicators of company growth; (1) cost of goods sold (with exponent $\beta = 0.16 \pm 0.03$), (2) assets ($\beta = 0.17 \pm 0.04$), and (3) property, plant and equipment ($\beta = 0.18 \pm 0.03$).

What is remarkable about equations (1) and (2) is that they govern the growth rates of a diverse set of firms. They range not only in their size but also in what they manufacture. The conventional economic theory of the firm is based on production technology, which varies from product to product. Conventional theory does not suggest that the processes governing the growth rate of car companies should be the same as those governing, for example, pharmaceutical or paper firms. Indeed, our findings are reminiscent of the concept of universality found in statistical physics, where different systems can be characterized by the same fundamental laws, independent of 'microscopic' details.

In statistical physics, scaling phenomena of the sort that we have uncovered in the sales and employee distribution functions are sometimes represented graphically by plotting a suitably 'scaled' dependent variable as a function of a suitably 'scaled' independent variable. If scaling holds, then the data for a wide range of parameter values are said to 'collapse' upon a single curve. To test the present data for such data collapse, we plot (Fig. 3) the scaled probability density $p_{\rm scal} \equiv \sqrt{2}\sigma(s_0)p(r \mid s_0)$ as a function of the scaled growth rates of both sales and employees $r_{\rm scal} \equiv \sqrt{2}[r - \bar{r}(s_0)]/\sigma(s_0)$. The data collapse upon the single curve $p_{\rm scal} = \exp(-\mid r_{\rm scal} \mid)$. Our results for (1) cost of goods sold, (2) assets, and (3) property, plant and equipment are equally consistent with such scaling.

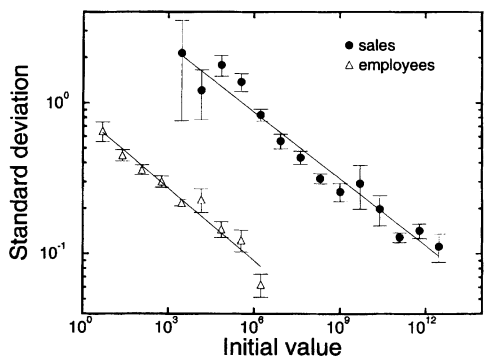

FIG. 2 Standard deviation of the one-year growth rates of the sales (circles) and of the one-year growth rates of the number of employees (triangles) as a function of the initial values. The solid lines are least-square fits to the data with slopes $\beta = 0.15 \pm 0.03$ for the sales and $\beta = 0.16 \pm 0.03$ for the number of employees. We also show error bars of one standard deviation about each data point. These error bars appear asymmetric as the ordinate is a log scale.

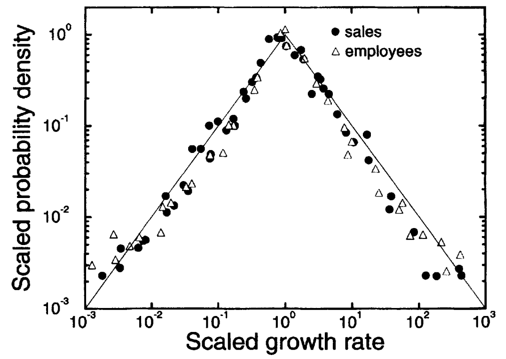

FIG. 3 Scaled probability density $p_{\rm scal} \equiv 2^{1/2}\sigma(s_0)p(r \mid s_0)$ as a function of the scaled growth rate $r_{scal} \equiv 2^{1/2}[r - \bar{r}(s_0)]/\sigma(s_0)$ of sales (circles). The values were rescaled using the measured values of $\bar{r}(s_0)$ and $\sigma(s_0)$. Also we show (triangles) the analogous scaled quantities for the number of employees. All the data collapse upon the universal curve $p_{\rm scal} = \exp(-\mid r_{\rm scal} \mid)$ (solid line) as predicted by equations (1) and (2).

The Gibrat model, which yields a log–normal distribution of the growth rates for sufficiently long time intervals, fails to explain the observed distribution of annual growth rates (even for intervals as long as 5 years, we find $p(r \mid s_0)$ does not obey a normal distribution.) There is, however, a simple dynamic process in which successive values of S_t are correlated that generates the observed tent-shaped distribution. Suppose each firm has a tendency to maintain a value S^*, which evolves only slowly in time and which can be interpreted as the minimum point of a 'U-shaped' average cost curve in conventional economic theory. This type of dynamics is similar to what is known in economics as regression towards the mean[27,28]. If the growth process has a constant 'back-drift', that is,

$$S_{t+\Delta t}/S_t = \begin{cases} k(1+\varepsilon_t) & \text{for } S_t < S^* \\ \frac{1}{k}(1+\varepsilon_t) & \text{for } S_t > S^* \end{cases} \quad (3)$$

where k is a constant larger than one, then the distribution of growth rates is the tent-shaped distribution equation (1) with a width proportional to $1/\ln k$ (ref. 29).

Our empirical findings of equation (2) are consistent with a hierarchical model of the internal structure of each firm. In zeroth-order approximation, suppose that a given company consists of independent units. If the unit's sales fluctuate with a standard deviation independent of s_0, then equation (2) follows with $\beta = 1/2$. The much smaller empirical value of β that we find indicates the presence of strong, positive correlations among the firm's units. We propose a model relying on a technology of management (which may be common across firms) as opposed to a technology of production; this model may lead to some insight into why the behaviour of apparently diverse firms follows a simple law.

Consider a tree-like hierarchical organization of a firm[17]. The root of the tree (that is, 'top of the pyramid') represents the head of the firm, whose policy is processed to the level beneath, and so on, until finally the $N = z^n$ lowest-level units take action; here z is the average number of links connecting the levels and n the average number of levels. The N lowest -level units have sales ξ_i and mean $\langle\xi\rangle$, so $S_0 = \sum_{i=1}^{N} \xi_i = N\langle\xi\rangle$.

Suppose that the head of the company suggests a policy with the intention to change the sales of each lowest-level unit by an amount $\Delta\xi$. If this policy were to be propagated through the hierarchy without any modifications, then the change in sales would be $\Delta S = N\Delta\xi = S_0\Delta\xi/\langle\xi\rangle$. Accordingly, $r = \ln[(S_0 + \Delta S)/S_0] = \ln[1 + \Delta\xi/\langle\xi\rangle]$, which is independent of S_0. It follows that $\beta = 0$.

More realistically, each unit is not only influenced by the policy of the head but also by other (external and internal) factors. An example is that different levels have different information. Managers at each level might deviate from decisions made higher up in the tree if other information suggests to them that another action is appropriate. Another reason for a modification of the policy is organizational failure, due either to poor communication or disobedience. For these reasons, we assume that each manager follows his supervisor's policy with a probability Π, while the probability $(1 - \Pi)$ imposes a new independent policy for his subunits. Straightforward calculation using methods described, for example, in ref. 30 yields equation (2) for $n \gg 1$, with exponent β given by the formula

$$\beta = \begin{cases} -\ln\Pi/\ln z & \text{if } \Pi > z^{-1/2} \\ 1/2 & \text{if } \Pi \leqslant z^{-1/2} \end{cases} \quad (4)$$

(for small n, equation (4) is still a good approximation—for example, for $n = 3$ and $z = 2$, the deviation from the value $\beta = 0.20$ is only 0.03). Equation (4) is confirmed in the two limiting cases: when $\Pi = 1$ (absolute control) $\beta = 0$, while for all $\Pi < 1/z^{1/2}$, decisions at the upper levels of management have no statistical effect on decisions made at lower levels, and $\beta = 1/2$. Moreover, for a given value of $\beta < 1/2$ the control level Π will be a decreasing function of z: $\Pi = z^{-\beta}$. For example, if we choose the empirical value $\beta \approx 0.15$, then equation (4) predicts the plausible result $0.9 \geqslant \Pi \geqslant 0.7$ for a range of z in the interval $2 \leqslant z \leqslant 10$.

Our central results, equations (1) and (2), constitute a test that any accurate theory of the firm must pass. These equations support the possibility[31] that the scaling laws used to describe complex systems comprised of many interacting inanimate particles (as in many physical systems) may be usefully extended to describe complex systems comprised of many interacting animate subsystems (as in economics). □

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Melting dynamics of a plasma crystal

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PLASMAS have long been regarded as the most disordered state of matter; nevertheless, a set of colloidal particles introduced into a charge-neutral plasma can spontaneously exhibit ordered crystalline structures[1,2]—so-called 'plasma crystals'. Such systems, which reach equilibrium very rapidly and can be easily tuned between their ordered and disordered states, are ideally suited for investigating the processes underlying the solid-to-liquid phase transition. Here we report the results of experiments on 'flat' plasma crystals (with thicknesses of only a few lattice planes) which suggest that the melting transition occurs through two fundamental intermediate stages. On melting, the crystal first enters a state characterized by islands of crystalline order, about which streams of particles flow. The crystalline regions then dissolve as the vibrational energy of the system increases, but this is accompanied by a temporary increase in orientational order before the system finally enters a disordered, liquid state. The unexpected 'vibrational' phase, characterized by enhanced orientational order, might arise as a consequence of the mixed two- and three-dimensional nature of the flat plasma crystals. Alternatively, it may indicate the existence of a new intermediate state in melting transitions more generally.

The forces controlling the structure and thermodynamics of plasma crystals are (1) Coulomb forces between the embedded particles and (2) neutral gas friction, which 'cools' the particles down to brownian motion. We consider Coulomb forces first. By colliding with electrons and ions in a plasma, micrometre-sized particles may obtain (negative) equilibrium charges, Q, of several thousand elementary charges, e (refs 1, 3). Equilibrium is reached in a fraction of a second. The plasma reorganizes itself locally in the electric field around the particle to neutralize its charge. The appropriate 'screening distance' is called the Debye length, λ_D; it depends on the plasma temperature, T, and the plasma density, n, as $\lambda_D = (kT/4\pi ne^2)^{1/2}$. Coulomb interaction between neighbouring particles (and therefore crystallization) can only occur if their separation is $\lesssim \lambda_D$. (This system may be considered to be comparable to a heavy atom whose positive core is neutralized by its electron cloud. The effective 'atom radius' is λ_D.) The second controlling force is neutral gas friction, which can provide the low particle kinetic energies required for crystallization. Experimental conditions for crystallization thus require a partially ionized plasma with a low- (room-) temperature neutral gas component. Such conditions can be produced easily in low-power radio-frequency discharges. For the experiment described here, we used krypton at room temperature and pressures between 0.1 and 0.5 mbar, a radio-frequency power of 0.8–2 W to give an ionization fraction from 10^{-7} to 10^{-6}, and monodispersive melamine/formaldehyde spheres of 6.9 μm diameter (see ref. 1 for details).

The parameters determining the thermodynamics of the plasma crystal system are (1) the Coulomb coupling parameter Γ. In the case of a screened Debye potential, $\Gamma \equiv (Q^2/\Delta kT)\exp(-\Delta/\lambda_D)$. This is the ratio of the Coulomb energy between two neighbouring particles (separated by a distance Δ) to their kinetic energy kT. (2) The ratio $\kappa = \Delta/\lambda_D$ of the lattice distance, Δ, to the Debye screening distance. Theoretical considerations[4,5] suggest that Coulomb crystallization may occur if $\Gamma \geqslant \Gamma_c = 172$ and $\kappa \lesssim 1$. (Γ_c is the critical value for crystallization obtained in Monte Carlo simulations of one-component-plasmas.) A decrease of Γ, or an increase of κ, leads to 'melting' of the crystalline structure. This can be initiated and controlled easily in radio-frequency discharge