New Exponent Characterizing the Effect of Evaporation on Imbibition Experiments

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We report imbibition experiments investigating the effect of evaporation on the interface roughness and mean interface height. We observe a new exponent characterizing the scaling of the saturated surface width. Further, we argue that evaporation can be usefully modeled by introducing a gradient in the strength of the disorder, in analogy with the gradient percolation model of Sapoval *et al.* By incorporating this gradient we predict a new critical exponent and a novel scaling relation for the interface width. Both the exponent value and the form of the scaling agree with the experimental results.

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Recently the growth of rough interfaces has witnessed a veritable explosion of theoretical and experimental results, partly fueled by the broad interdisciplinary aspects of the subject [1]. Much attention has focused on measuring the roughness exponent α, defined by the power law dependence upon the observation length scale ℓ of the width $w(\ell)$. Simulations on discrete models provide exponents in agreement with the predictions of phenomenological continuum theories [2]. However, experimental studies find exponents significantly larger than the predictions of theory—for example, for dimension $d = (1 + 1)$, theory predicts $\alpha = 1/2$ but experiments show $\alpha \simeq 0.63-0.8$ [3,4]. Moreover, experimental studies frequently detect a crossover in $w(\ell)$ to a different behavior above some characteristic length $\ell_\times$. It is currently believed that the anomalously large values of the exponent α are due to quenched pinning disorder [1,4,5], however, a completely satisfactory explanation of the experimentally determined crossover length has not yet been found.

Here we present imbibition experiments that probe the effect on the growth process of the evaporation rate and suspension concentration. We find that the scaling of the interface width changes with the evaporation rate and is characterized by a new exponent γ. We also present a model, inspired by the model of Ref. [4], that predicts the experimentally observed value of the new exponent characterizing the crossover effect.

The key ingredient in the model is to allow for a gradient in the density of the pinning cells, which results in the stopping of the interface. Moreover, the detailed investigation of the scaling properties of the model provides us with a new scaling law and new critical exponents. Using this scaling law, we find good data collapse and scaling exponents that agree with the values determined analytically and numerically.

In our experiments, paper — the "disordered medium" — is dipped into a reservoir filled with a colored suspension (coffee, ink) and the propagating wetting front is observed. The wetting front reaches a critical height, h_c, above the level of the liquid, and stops propagating when the evaporation of the liquid induces the pinning of the interface by the inhomogeneities of the paper. We digitize the rough boundary between colored and uncolored areas and measure a roughness exponent $\alpha \simeq 0.63$ [4].

Although the experiments are straightforward, their explanation in physical terms is less so. At microscopic length scales, paper is an extremely disordered substance, formed by long fibers that are randomly distributed and have random connections among one another. The wetting fluid propagates in these fibers mainly due to capillary forces, but the random nature of the fiber network and the particles in the suspension provide constant obstacles for the fluid flow [6]. As we depart from the water source, evaporation is constantly decreasing the fluid pressure, making it more and more difficult for the fluid to overcome these microscopic "obstacles." At the critical height, the fluid pressure balances the effect of the pinning obstacles and the fluid stops propagating [7].

We anticipate, on physical grounds, that the smaller the evaporation, the larger the critical height will be. To check this intuition we repeated our experiments in environments with different rates of evaporation. As we decreased the evaporation rate, the height reached by the interface increased sharply. These effects can be observed in the pinned interfaces obtained experimentally [Fig. 1(a)].

While the above results can be understood from microscopic considerations, the effect of the evaporation or of the suspension density on the scaling of the interface roughness is nontrivial.

To understand these results, we propose a model to describe the formation of the pinned interface in a disordered medium. In $(1 + 1)$ dimension, we model the pinning obstacles by blocking, in a lattice of horizontal size L, a fraction $p(h)$ of the cells in each horizontal row, where h is the height from the bottom of the lattice. We start, at $t = 0$, from a horizontal line of wet cells at the

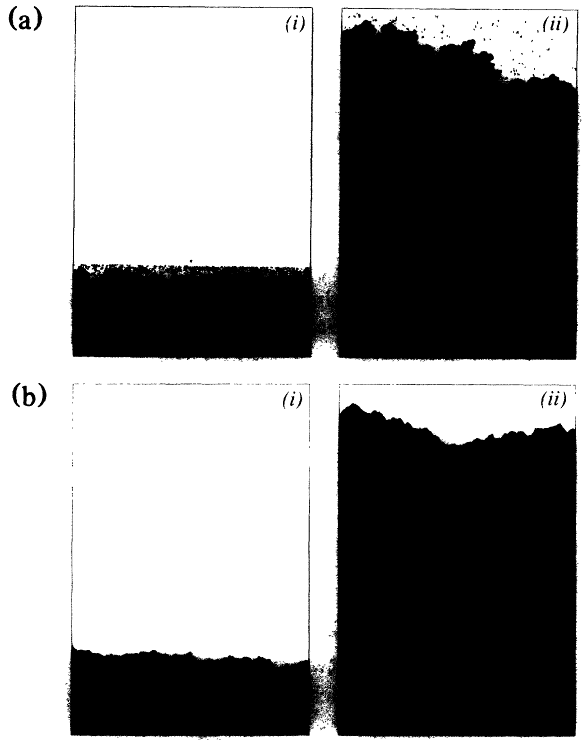

FIG. 1. Photographs of pinned interfaces in the following: (a) Imbibition experiments with coffee and paper towels for (i) high evaporation rate: $(\nabla p)_{\text{exp}} = 0.94 g_0$, and (ii) low evaporation rate: $(\nabla p)_{\text{exp}} = 0.25 g_0$. Here g_0 is the undetermined multiplicative constant discussed in the text. (b) Simulations of the model, with $L = 256$, for different values of the gradient: (i) $\nabla p = 2^{-8}$ and (ii) $\nabla p = 2^{-10}$. Readily apparent from these photographs is the increase in both the final heights and widths of the interface with the decrease of the gradient.

bottom edge of the lattice. At time $t+1$ we wet all unblocked cells which are *nearest neighbors to the wet region* at time t. We also apply the rule that every cell, *blocked or not*, below a new wet cell becomes wet as well (Fig. 2). The motivation for this rule is the experimental observation that the wet region is, at least at macroscopic length scales, nearly free of dry islands.

If $p(h) = p_0$, this model generates an interface which propagates with a constant (p-dependent) velocity if $p_0 < p_c \simeq 0.47$, and becomes pinned by a *directed percolation cluster* that spans the system at $p_0 \geq p_c$ [4,5]. However, although the actual disorder in the paper *is not height dependent*, its *effect* in pinning the propagation of the fluid *is increasing with height*, due to the decrease in the fluid pressure. The most physical assumption is an exponential decrease of the fluid pressure or, equivalently, of the driving force. This will lead to an "effective" increase in the density of pinning obstacles [7] as we depart from the reservoir, i.e., $p \equiv p(h)$. Hence

$$p(h) - p_0 \propto 1 - e^{-h/h_0}. \tag{1}$$

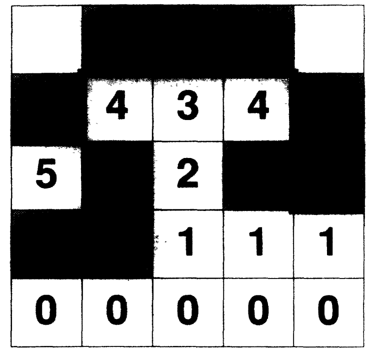

FIG. 2. Example of the time evolution of the model for a very small lattice ($L = 5$). Here, grey squares represent blocked cells and white squares represent unblocked cells. The numbered cells are wet. The numbers indicate at which time step the cells first become wet. At $t = 4$, we wet the cells at the left and at the right of the cell numbered 3. Also, in the same time step we wet the cells below those two, regardless of the fact that they were previously blocked. Similarly, at $t = 5$, we are able to wet cells in the first column from wet cells in the second column that were, at some earlier time, blocked cells. The heavy line indicates the pinned interface.

If $h \ll h_0$, we can write

$$p(h) - p_0 \propto h_0^{-1} h \propto (\nabla p) h. \tag{2}$$

Hence, in this limit, we find a constant *nonzero gradient* in the density of pinning obstacles.

The presence of the gradient ∇p changes the width of the pinned interface [Fig. 1(b)] and its scaling form (Fig. 3). Our simulations show that for observation scales ℓ much smaller than some characteristic crossover length $\ell_\times$, the saturated width behaves as $w \sim \ell^\alpha$, but for $\ell \gg \ell_\times$, the width saturates at a value w_{sat} that depends upon the gradient as

$$w_{\text{sat}} \sim (\nabla p)^{-\gamma}. \tag{3}$$

This behavior can be expressed by a scaling law of the form

$$w(\ell, \nabla p) \sim \ell^\alpha f(\ell/\ell_\times), \tag{4a}$$

$$\ell_\times \sim (\nabla p)^{-\gamma/\alpha}. \tag{4b}$$

The scaling function $f(u)$ satisfies $f(u \ll 1) \sim$ const and $f(u \gg 1) \sim u^{-\alpha}$. Our simulations (see Fig. 3) for a system of size $L = 16\,384$ yield the exponents

$$\alpha_{\text{sim}} = 0.63 \pm 0.02, \quad \gamma_{\text{sim}} = 0.52 \pm 0.02. \tag{5}$$

We remark that the validity of the scaling law (4) and the values of the exponents do not depend on the exact form of $p(h)$ but only on the value of $\nabla p(h)$ at h_c [8].

The value of α can be understood from the mapping to directed percolation, since the conditions for a complete

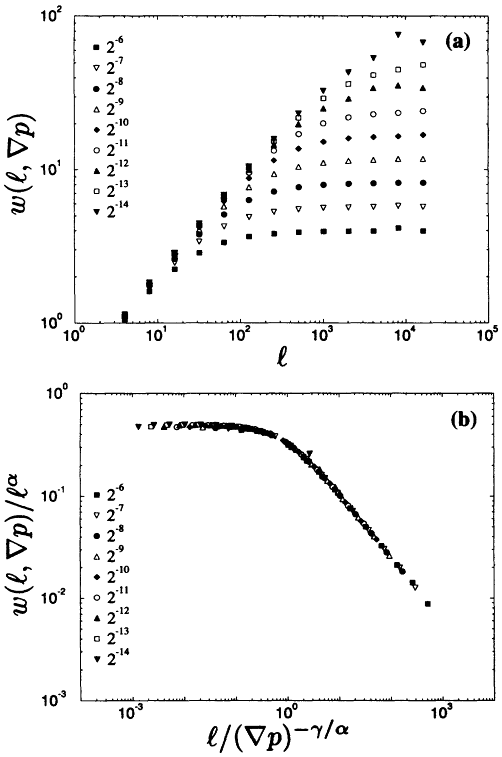

FIG. 3. The simulation results for the width $w(\ell, \nabla p)$ of the pinned interface. (a) The widths for several values of the gradient (averaged over 512 runs for each value of the gradient). (b) The same simulation results, plotted in the scaling form of Eq. (3), using the values of the exponents from Eq. (4).

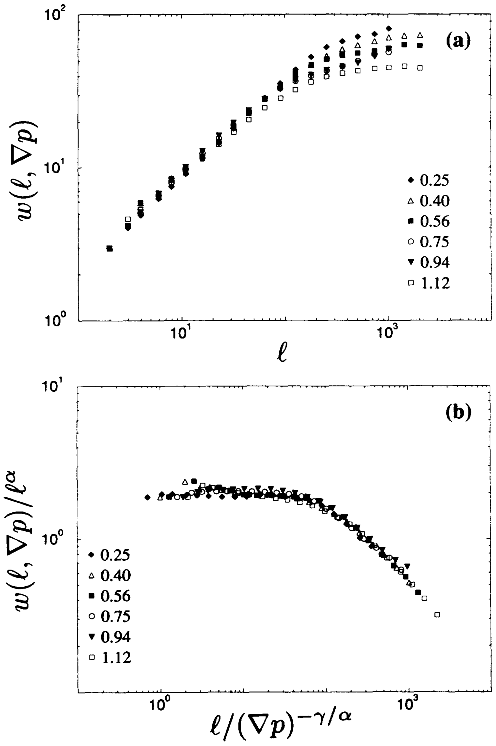

FIG. 4. The experimental results for the width $w(\ell, \nabla p)$ of the pinned interface. (a) The widths for several values of the gradient (in units of g_0). The values of the gradients were calculated as described in the text; the error in these values is smaller than 10%. The widths were corrected by a multiplicative factor to make them coincide for the smallest ℓ. (b) The same experimental results, plotted in the scaling form of Eq. (3), using the values of the exponents from Eq. (10).

pinning of the interface do not change from the models of Refs. [4,5]. In directed percolation, the size of a cluster is characterized by a longitudinal correlation length $\xi_{\|}$ and a transverse correlation length $\xi_{\perp}$ that, near p_c, behave as

$$\xi_{\|} \sim |p_c - p|^{-\nu_{\|}}, \quad \xi_{\perp} \sim |p_c - p|^{-\nu_{\perp}}. \tag{6}$$

The roughness exponent is related to the exponents of directed percolation as [4,5]

$$\alpha = \nu_{\perp}/\nu_{\|}. \tag{7}$$

Using the known values of $\nu_{\perp}$ and $\nu_{\|}$ [9] in relation (7) we predict $\alpha = 0.633 \pm 0.001$, in agreement with our simulation result (5).

The exponent γ can be related to $\nu_{\perp}$ theoretically. A point of the interface, at distance $w_{\rm sat}$ of the critical height, is pinned by a directed percolation cluster if the transverse size of that cluster is of order $\xi_{\perp}(p)$. At that point we have $p = p(h_c \pm w_{\rm sat}) \approx p_c \pm w_{\rm sat} \nabla p$. Therefore, using Eq. (6) we find [8,10]

$$w_{\rm sat} \sim \xi_{\perp}(p) \sim |p_c - (p_c \pm w_{\rm sat} \nabla p)|^{-\nu_{\perp}},$$
$$w_{\rm sat} \sim |w_{\rm sat} \nabla p|^{-\nu_{\perp}}. \tag{8}$$

From Eqs. (3) and (8) follows

$$\gamma = \nu_{\perp}/(1 + \nu_{\perp}). \tag{9}$$

Since $\nu_{\perp}$ is known accurately [9], Eq. (9) predicts $\gamma = 0.523 \pm 0.001$, in excellent agreement with our simulation result (5).

Without the gradient, the interface has critical behavior only if we tune p to p_c. However, with the gradient the interface always stops at the critical height h_c. This critical height can be calculated from the condition $p(h_c) = p_c$. Thus from (2) we obtain

$$h_c \sim (\nabla p)^{-1}, \tag{10}$$

i.e., the height reached by the wetting fluid is inversely proportional to the gradient in the disorder.

The experimental data presented in Fig. 4(a) remarkably resemble the data obtained for the model. However, without knowing the actual value of the gradient in the experiments, it is not possible to check the validity of the scaling law (4) experimentally. Nonetheless, measuring the critical height in the experiments and using Eq. (10),

TABLE I. Critical probability and exponents for dimension (1 + 1) and (2 + 1), calculated from the simulations of the present model.

Dim.	(1 + 1)	(2 + 1)
p_c	0.47 ± 0.03	0.75 ± 0.03
α	0.63 ± 0.02	0.43 ± 0.04
γ	0.52 ± 0.02	0.32 ± 0.02
$\nu_\perp$	1.09 ± 0.08	0.47 ± 0.04
$\nu_\parallel$	1.7 ± 0.1	1.1 ± 0.1

we are able to estimate ∇p, the gradient in the "effective disorder," for the experiments, up to a multiplicative constant. Using these experimentally determined values of ∇p, we rescale the results obtained for the width according to the scaling law (4). In Fig. 4(b) we show this rescaling, where we used

$$\alpha_{\mathrm{exp}} = 0.65 \pm 0.05, \quad \gamma_{\mathrm{exp}} = 0.49 \pm 0.05. \tag{11}$$

The experimental values of both exponents agree well with the results obtained from the simulations (Table I) and with the theoretical predictions based in known results from directed percolation.

The generalization of the model to $d = (2+1)$ is straightforward, and in this case the pinned interface can be mapped to directed surfaces [4], a percolation problem that has not been thoroughly investigated. We simulated the model for a 512×512 system; the critical exponents that give the best data collapse are

$$\alpha_{\mathrm{sim}} = 0.43 \pm 0.04, \quad \gamma_{\mathrm{sim}} = 0.32 \pm 0.02. \tag{12}$$

From these results, we calculate the exponents characterizing the transverse and longitudinal correlation lengths for the directed surfaces problem, obtaining

$$\nu_\perp = 0.47 \pm 0.04, \quad \nu_\parallel = 1.1 \pm 0.1. \tag{13}$$

In summary, we have performed imbibition experiments to study the effect of evaporation on interfacial phenomena. We have also developed a model that incorporates evaporation by introducing a gradient in the density of pinning cells [7]. The model provides insight into three previously unexplained aspects of imbibition experiments: (i) The interface always stops growing, after some finite time. Because of the gradient, the wetting interface only moves until it reaches a critical concentration of pinning cells. This gradient in pinning cells arises from the balance between the evaporation of the fluid and the capillary forces tending to move it along the paper. (ii) The final height of the interface, h_c, increases when the evaporation is reduced, due to the smaller *effective* gradient in the pinning disorder. (iii) A new exponent γ was found characterizing the dependence on the gradient of the saturation width and the characteristic length $\ell_\times$. Good agreement was found between experimental, analytical, and simulation values of the exponents.

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[6] In this work we refer to evaporation as the origin of the gradient. However, the density of the suspension has a similar role in changing the balance between capillary forces and pinning obstacles by also setting the value of the gradient. Indeed, our experiments suggest that the same model accounts for not only evaporation effects, but also suspension density effects (cf. Fig. 4).

[7] We stress the fact that the actual disorder in the paper is height independent. However, due to the decrease in capillary pressure with height, the *effect* of the inhomogeneities is increasing with height. In the model we take account of this changing balance between capillary pressure and strength of the inhomogeneities by introducing a gradient in the "effective" density of pinning obstacles.

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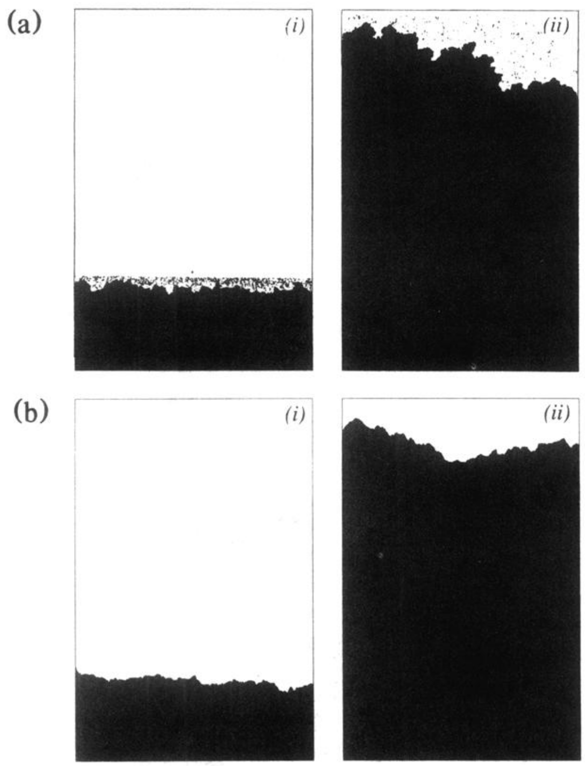

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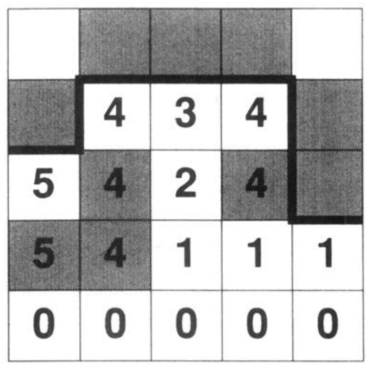

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