Araujo et al. Reply: Cornell [1] presents quantitative discrepancies between the results of his simulations and those reported in [2], from which he concludes that the reaction front between initially separated diffusing reactants can be described asymptotically by a single length scale. Crucial to his conclusion is his contention that the fluctuations in the position of the midpoint coordinate between the leftmost B particle (LMB) and the rightmost A particle (RMA) eventually grow in time as \( t^{1/4} \).

We believe that this contention cannot be valid: If instead of considering the midpoint we consider a “ghost” diffusing particle confined between the LMB and the RMA, this particle can be considered to have “almost” a hard core interaction with the reactants A and B, the exception being when a reaction occurs. In the absence of reaction, the ghost particle would satisfy Richards’ law for hard core diffusion \( \xi_{\text{nonreactive}} \sim (Dt)^{1/4}/c_0^{1/2} \) [3]. In the presence of reactions, for any \( c_0 \) in Richards’ law, we can wait a finite time \( t_0 \), beyond which the concentration in the vicinity of the ghost particle, and within the diffusive correlation length generated thereafter, is much smaller than \( c_0 \). Then, from \( t_0 \) on the characteristic length \( \xi_{\text{reactive}} \) describing the fluctuations of the ghost particle will satisfy \( \xi_{\text{reactive}} > (Dt - t_0)^{1/4}/c_0^{1/2} \). Since this is true for any \( c_0 \), we can conclude that, asymptotically, \( \xi_{\text{reactive}} \sim t^\beta \) with \( \beta \) strictly larger than \( 1/4 \). Indeed, an argument \( \text{à la} \) Richards (in which the average number of particles displaced by the ghost particle is matched by the fluctuation in the number of particles within a diffusion length of the ghost particle) for the reactive case, reproduces the value \( \beta = 3/8 \), which was obtained by a different argument in [2].

Motivated by the work of [1], we thoroughly reexamined the simulation procedure upon which our numerical values were obtained. We found that indeed upon taking much higher statistics at larger times some of the properties of the system appear to depart from the results reported in [2]. Specifically, we find that for the times under consideration the fluctuations in the coordinate of the midpoint appear to be described by a length scale that grows in time with an exponent \( \beta \) smaller than \( 3/8 \). Nevertheless, we find no evidence that the \( \beta \) tends to \( 1/4 \), as claimed in [1].

Moreover, in our extended simulations, the “multiscaling” behavior observed originally still appears to be present, although the range of exponents is smaller \([0.27 \leq \alpha(q) \leq 0.31 \sim \beta]\).

Thus, the discrepancies, although important, do not appear to invalidate our argument that in confined systems, the midpoint dynamics and the reaction front are strongly correlated, and that the properties of the reaction front in these systems actually stem from these correlations. Hence any conclusions drawn from steady state analysis of the system are of limited value, since the main contribution to the shape of the reaction front—the fluctuations of the midpoint—is arrested in this situation. Actually, again drawing an analogy with hard core diffusion, the dynamics of a “marked” hard core particle do not ever reach a steady state.

We also contend, for reasons developed in [2], that the correlations between the midpoint and the reaction front are also at the root of the essential differences between quasi-one-dimensional systems and higher dimensional systems where mean field theory applies [4].

Finally, we note that the length scale \( \xi \sim t^{3/8} \), obtained in [1] roughly as the length at which the fluctuations of the flux of one species are balanced by the concentration profile of the other, also appears as the fundamental length scale in “the true asymptotic regime” of the reaction front according to a recent analytical treatment of the problem [5]. The multiscaling behavior is also in qualitative agreement with RG results [6] for the moments of the reaction front. It is, thus, possible that the simulations performed so far might still be probing transient behaviors. Thus we conclude that this problem is still a long way from being understood, and more work is certainly called for.

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