Diffusion

Diffusion is the random migration of molecules or large particles that usually arises from thermal energy. Fundamental dynamical processes in nature such as ionic transport through cell membranes, molecular transport in liquids, hydrogen diffusion in metals, ionic diffusion in glassy material, transport of electrons and excitons in condensed phases, atomic and molecular diffusion on surfaces, surface growth, motion of microorganisms, dynamics of populations as well as many other examples have all been described in terms of diffusion [1–7].

In most cases, the motion of a diffusing species is characterized by a mean-squared displacement that apart from very short times increases linearly with time,

$$\langle r^2(t) \rangle = 2dDt, \tag{1}$$

where d is the spatial dimension and D is the diffusion constant. More specifically, the diffusive motion is characterized by the probability $P(r,t)$ that the species is in a unit volume around r at time t when starting at $t = 0$ from the origin. Fick showed that for noninteracting species $P(r,t)$ satisfies the *diffusion equation*

$$\frac{\partial P(r,t)}{\partial t} = D \nabla^2 P(r,t), \tag{2}$$

and is a Gaussian

$$P(r,t) = (2\pi Dt)^{-d/2} \exp(-dr^2/4Dt). \tag{3}$$

Equations (1) and (4) provide almost everything one needs to know about classical diffusion, where interactions among the particles and with the substrate can be neglected. A useful model for diffusion is the *random-walk model*, where a species advances in one unit of time to a randomly chosen site nearby.

For charged particles, the diffusion constant is related to the conductivity σ of the medium by the Nernst-Einstein relation

$$\sigma \cong (ne^2/kT)D \tag{4}$$

where n is the density of particles with charge e, k is the Boltzmann constant and T is the absolute temperature.

There is a growing body of evidence that classical diffusion does not hold generally [5–11]. Rather, anomalous laws of diffusion exist, both slower and enhanced relative to classical diffusion,

$$\langle r^2(t)\rangle \sim t^{\alpha}, \qquad \alpha \neq 1. \tag{5}$$

Cases with $\alpha < 1$, known as the dispersive transport regime, have been the topic of many theoretical and experimental studies and have been attributed to random walks on random self-similar structures or to temporal disorder, both having underlying scale-invariant properties. Prominent examples for dispersive transport are diffusion in disordered materials (amorphous semiconductors, polymer networks, gels, and ionic glasses) and defect diffusion in relaxation studies. For random walks on fractals one finds that

$$\alpha = 2/d_w, \qquad d_w > 2. \tag{6}$$

where $d_w = 2d_f/d_s$ is the fractal dimension of a random walk, d_f the fractal dimension, and d_s the spectral dimension. In addition, Eq. (3) is no longer valid but substituted by a stretched Gaussian.

The enhanced diffusion regime, $\alpha > 1$ in Eq. (5) has been also studied extensively. Probably the most known example of enhanced diffusion is Richardson's observation of turbulent diffusion, where $\alpha \approx 3$. Other examples include diffusion in elongated micelles, transport of high density excitons, recent suggestions on the analysis of director fluctuations in liquid crystals in the nematic phase, tracer diffusion in underground water, and the analysis of DNA nucleotide sequences.

To model enhanced diffusion, Lévy generalized the random-walk model to what is now called Lévy flights. While in random walks the step length is bounded, Lévy considered a motion taking steps of size $\{x_i\}$ from the probability density $p(x)$

$$px) \sim |x|^{-1-\gamma}, \qquad \text{for} \quad x >> 1, \tag{7}$$

that is the moments $\langle |x|^\delta \rangle$ are finite for $\delta < \gamma$ and are infinite for $\delta \geq \gamma$. Modifications to space-time coupled behavior, called Lévy walks, have been useful in understanding a broad range of enhanced dynamical properties in fluid mechanics, interfacial diffusion, and polymers.

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