Cohen, Erez, Ben-Avraham, and Havlin Reply: In our Letter [1] we studied the resilience of scale-free networks to intentional attack (deletion of the most highly connected nodes). Our main result is a formula for $p_c$ — the fraction of most connected sites that must be removed before the network collapses — which follows from Eqs. (8) and (11) [1]:

$$\frac{2}{3} - \frac{a}{3 - \alpha} m(\frac{2}{3} - \alpha - 1).$$

(1)

This was derived under the assumption that $P(k)$, the probability that a site has $k$ connections, is modeled by the continuous distribution

$$P(k) = ck^{-\alpha}, \quad m \leq k \leq K,$$

(2)

where $c$ is a normalization constant, and $m$ and $K$ are lower and upper cutoffs for the site connectivity, respectively. In practice, though, a site may have only an integer number of connections. Indeed, in our simulations [1] we have used the discrete distribution

$$P_1(k) = \int_{k-1/2}^{k+1/2} P(q) \, dq.$$ (3)

The analytical formula of Eq. (1) provides an excellent approximation to results from simulations performed with the distribution $P_1(k)$; see Fig. 1 in [1].

The Comment’s [2] main claim is that in [1] we did not compare our results to the discrete distribution [3,4]:

$$P_{11}(k) = \frac{k^{-\alpha}}{\xi(\alpha)}, \quad k = 1, 2, \ldots.$$ (4)

Following our theory, the authors of the Comment show that $p_c$, for the distribution $P_{11}(k)$, is given by the solution to the set of equations:

$$\sum_{k=1}^{k(p_c)} k^{-\alpha} = \frac{\xi(\alpha - 1)}{\xi(\alpha)} + \sum_{k=1}^{k(p_c)} k^{1-\alpha},$$

(5a)

$$p_c = 1 - \sum_{k=1}^{k(p_c)} k^{-\alpha}/\xi(\alpha).$$ (5b)

Because the authors of the Comment regard the distribution $P_{11}(k)$ as “genuine” compared to $P_1(k)$, they view with alarm the differences in $p_c$ obtained from the two distributions.

We observe that (a) $P_1$ and $P_{11}$ are equal, asymptotically, in the limit of large $k$ and (b) the differences are most pronounced for $k = m$, where $P_1(1)$ is quite smaller than $P_{11}(1)$. The difference in $p_c$ between our approach and Ref. [4] is mainly due to the values of $P(k)$ for small $k$ and is not related to the type of approximation, continuous or discrete. More details will be forthcoming [5].

Moreover, we strongly disagree that, in the context of the Internet, $P_{11}$ is more original than $P_1$. While it has been firmly established that $P(k) \sim k^{-\alpha}$ for large $k$ [6], which is valid for both $P_1(k)$ and $P_{11}(k)$, the distribution for small $k$ has not been explored. In this limit, the distribution is most fluid, due to computers connecting and detaching from the net. Our aim in [1] has been merely to explore the effect of the scale-free tail (at large $k$).

Surely, the simplicity of Eq. (1), vs. à-vis Eqs. (5), more than makes up for any conceivable aesthetic advantage of $P_{11}$ over $P_1$. The use of the distribution $P_1$ (and its continuous analog) is more than worthwhile.

Reuven Cohen, Keren Erez, Daniel Ben-Avraham, and Shlomo Havlin

1 Minerva Center and Department of Physics
Bar-Ilan University
Ramat-Gan, Israel
2 Department of Physics
Clarkson University
Potsdam, New York 13699-5820

Received 14 August 2001; published 31 October 2001
DOI: 10.1103/PhysRevLett.87.219802
PACS numbers: 89.20.Hh, 02.50.Cw, 64.60.Ak, 89.75.Hc

*Email address: cohenr@shoshi.ph.biu.ac.il
[3] Note that the Comment specializes to lower cutoff $m = 1$.