Cohen, Erez, ben-Avraham, and Havlin Reply: In our Letter [1] we studied the resilience of scale-free networks to intentional attack (deletion of the most highly connected nodes). Our main result is a formula for p_c—the fraction of most connected sites that must be removed before the network collapses—which follows from Eqs. (8) and (11) [1]:

$$p_c^{\frac{2-\alpha}{1-\alpha}} - 2 = \frac{2-\alpha}{3-\alpha} m\left(p_c^{\frac{3-\alpha}{1-\alpha}} - 1\right). \tag{1}$$

This was derived under the assumption that $P(k)$, the probability that a site has k connections, is modeled by the *continuous* distribution

$$P(k) = ck^{-\alpha}, \qquad m \leq k \leq K, \tag{2}$$

where c is a normalization constant, and m and K are lower and upper cutoffs for the site connectivity, respectively. In practice, though, a site may have only an *integer* number of connections. Indeed, in our simulations [1] we have used the *discrete* distribution

$$P_{\mathrm{I}}(k) = \int_{k-1/2}^{k+1/2} P(q)\, dq. \tag{3}$$

The analytical formula of Eq. (1) provides an excellent approximation to results from simulations performed with the distribution $P_{\mathrm{I}}(k)$; see Fig. 1 in [1].

The Comment's [2] main claim is that in [1] we did not compare our results to the discrete distribution [3,4]:

$$P_{\mathrm{II}}(k) = k^{-\alpha}/\zeta(\alpha), \qquad k = 1, 2, \ldots. \tag{4}$$

Following our theory, the authors of the Comment show that p_c, for the distribution $P_{\mathrm{II}}(k)$, is given by the solution to the set of equations:

$$\sum_{k=1}^{\tilde{K}(p_c)} k^{2-\alpha} = \zeta(\alpha - 1) + \sum_{k=1}^{\tilde{K}(p_c)} k^{1-\alpha}, \tag{5a}$$

$$p_c = 1 - \sum_{k=1}^{\tilde{K}(p_c)} k^{-\alpha}/\zeta(\alpha). \tag{5b}$$

Because the authors of the Comment regard the distribution $P_{\mathrm{II}}(k)$ as "genuine" compared to $P_{\mathrm{I}}(k)$, they view with alarm the differences in p_c obtained from the two distributions.

We observe that (a) P_{I} and P_{II} are equal, asymptotically, in the limit of large k and (b) the differences are most pronounced for $k \approx m$, where $P_{\mathrm{I}}(1)$ is quite smaller than $P_{\mathrm{II}}(1)$. The difference in p_c between our approach and Ref. [4] is mainly due to the values of $P(k)$ for small k and is not related to the type of approximation, continuous or discrete. More details will be forthcoming [5].

Moreover, we strongly disagree that, in the context of the Internet, P_{II} is more original than P_{I}. While it has been firmly established that $P(k) \sim k^{-\alpha}$ for large k [6], which is valid for both $P_{\mathrm{I}}(k)$ and $P_{\mathrm{II}}(k)$, the distribution for small k has not been explored. In this limit, the distribution is most fluid, due to computers connecting and detaching from the net. Our aim in [1] has been merely to explore the effect of the scale-free tail (at large k).

Surely, the simplicity of Eq. (1), *vis-à-vis* Eqs. (5), more than makes up for any conceivable aesthetic advantage of P_{II} over P_{I}. The use of the distribution P_{I} (and its continuous analog) is more than worthwhile.

Reuven Cohen,[1,*] Keren Erez,[1] Daniel ben-Avraham,[2] and Shlomo Havlin[1]
[1]Minerva Center and Department of Physics
Bar-Ilan University
Ramat-Gan, Israel
[2]Department of Physics
Clarkson University
Potsdam, New York 13699-5820

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*Email address: cohenr@shoshi.ph.biu.ac.il

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