Comment on the Aharony-Stauffer Conjecture

In a recent Letter[1] Aharony and Stauffer (AS) suggested that for any fractal aggregate for which the fractal dimension $d_f < 2$, the diffusion exponent d_w and the fracton dimension $\bar{\bar{d}}$ will be given as

$$d_w = d_f + 1, \quad \bar{\bar{d}} = 2d_f/(d_f+1). \tag{1}$$

Their arguments are based mainly on the assumption that the front of the random walk (RW) on the fractal, G, is a spherical cut of the fractal[2], that is, $G \approx R^{d_f-1}$. This assumption, speculated by AS to apply to all fractals, implies that for any fractal (random or exact) Eqs. (1) will be valid.

In this Comment I present two examples for which the above assumption is not correct and therefore Eq. (1) is not valid. I begin with the case of aggregates for which loops can be neglected. In this case it was shown[3] that since loops can be neglected

$$d_w = d_f(1+1/d_l), \quad \bar{\bar{d}} = 2d_l/(d_l+1), \quad d_w^l = d_l + 1 \tag{2}$$

the exponents $\bar{\bar{d}}$ and d_w^l are independent of d_f. The exponent d_l is the chemical dimension defined[4] by $S \approx l^{d_l}$ where S is the mass included within a chemical distance. (The length of the minimum path between two sites is the chemical distance between these sites.) The chemical diffusion exponent d_w^l is defined by $l^{d_w^l} \approx t$. The physical interpretation of Eq. (2) is that the front of the RW is a cut of the fractal in the chemical distance, $G \approx l^{d_l-1}$, and not in the geometrical distance R as assumed by AS in order to derive Eq. (1). Only for the special case $d_l = d_f$, which is unusual, as will be discussed below, will Eq. (2) reduce to Eq. (1). For the general case $d_l < d_f$ (not equal) one finds from Eq. (2)

$$d_w > d_f + 1 \quad \text{and} \quad \bar{\bar{d}} < 2d_f/(d_f+1), \tag{3}$$

which contradicts with the AS conjecture. Moreover, for any cluster the inequality $d_l \leq d_f$ is valid. This is due to the fact that the geometrical distance cannot be larger than the chemical distance l and that $R \approx l^{d_l/d_f}$. However, the case $d_l = d_f$ is unusual since this means that the path scales linearly with the geometrical distance as a straight line.[5] Thus we conclude that for any cluster for which loops are irrelevant and $d_l < d_f$ (not equal) the front of the walk is not a spherical geometrical cut of the fractal and therefore Eq. (1) is not valid.

The above discussion may apply to aggregates like lattice animals, diffusion-limited aggregation, and cluster-cluster aggregates for which it is reasonable to assume that loops are not relevant for dynamic properties, since it is accepted that loops may be ignored when calculating static properties. Indeed, for the case of lattice animals, in $d=2$, it was found recently[3] that $d_l/d_f = 0.83$, that is $d_l < d_f$, and the diffusion data were found to be in agreement with Eq. (2) rather than Eq. (1).[5]

The case of clusters for which loops are relevant is not as clear. In this case Eqs. (2) are not valid but serve as an upper limit for d_w and lower limit for $\bar{\bar{d}}$. This is due to the fact that the effect of loops is to decrease the diffusion (or the resistivity which is related to it through the Einstein relation). However, there is an example of an ordered exact fractal (which may or may not teach us about random fractals) for which loops are relevant and the front of the RW is not a cut of the fractal. This is the case of the Sierpinski gasket which has been solved exactly.[6] In this case it can be easily shown that $d_f = d_l = \ln 3/\ln 2$ $(d=2)$ and $G \approx R^{\ln(9/5)/\ln 2} \neq R^{d_f-1} = R^{d_l-1} = R^{\ln(3/2)/\ln 2}$. Thus in this case the front of the walk is not a spherical cut of the fractal in both the R space and the l space as assumed in Ref. 1 and thus Eq. (1) is not valid.

To summarize I have presented two examples, clusters without loops and Sierpinski gasket, for which the AS assumption that the front of the walk is a spherical cut of the fractal is not valid. The case of percolation clusters is still an open question.

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[2] S. Alexander, in *Percolation Clusters and Structures*, edited by J. Adler, G. Deutcher, and R. Zallen, Annals of the Israel Physical Society (Hilger, London, 1983).

[3] S. Havlin, Z. Djordjevic, I. Majid, H. E. Stanley, and G. W. Weiss, Phys. Rev. Lett. **53**, 178 (1984).

[4] S. Havlin and R. Nossal, J. Phys. A **17**, L427 (1984).

[5] Although $d_l = d_f$ is unusual, it may occur in screened-growth aggregates for which P. Meakin, F. Leyvraz, and H. E. Stanley (to be published) find that both Eqs. (1) and (2) are good approximations.

[6] Y. Gefen, A. Aharony, B. B. Mandelbrot, and S. Kirkpatrick, Phys. Rev. Lett. **47**, 1771 (1981).