differences in the conception sex ratio related to individual phenotypic variation may require favourable environmental conditions, whereas differences in the susceptibility of male and female fetuses to nutritional stress may generate population-wide trends in annual birth sex ratios. In the Rum red deer population, the action of one mechanism swamped the other within about two generations. This may explain why general trends in sex-ratio variation have been so difficult to detect.

Methods

Since 1971, life history data have been collected on individual red deer (Cervus elaphus) in the North Block of the Isle of Rum, Scotland, an area of about 12 km² (ref. 18). We take as our measure of density the number of females of more than one year old in the study area; since the cessation of culling in 1973, density has risen from 57 to 178. Females produce at most one calf per year throughout their breeding lifespan, whereas male reproductive success shows greater variance; adult male weight is 1.7 times that of females, and male calves are heavier at birth18. All animals in the study population are individually recognizable; daily monitoring of the population during the calving season shows whether or not each female calved in a given year, and, if so, the sex of her offspring. An age-corrected dominance rank for each female, ranging from 0 to 1, is calculated from observations of interactions between pairs of individuals, as described elsewhere. Females are also classified as to whether or not they reared a calf the previous year that survived to six months. Average temperature and total rainfall in the following periods were considered: August to October, November to January, February to March. None of the weather variables showed consistent change over the study period.

We report two forms of statistical analysis. The proportion of males born each year (referred to as the annual birth sex ratio) was related to density and weather measures using simple linear regression (normality of errors was assessed; 1973 was excluded owing to incomplete data collection). The probability that an individual calf was male and the probability that a female gave birth in a given year were analysed using generalized linear mixed models28, with restricted maximum likelihood estimation of variance components. Just as generalized linear models allow the extension of general linear models to data where the errors are not normally distributed, GLMMs allow similar extensions to the conventional mixed model case where the response variable is determined by both random and fixed effects. In both models, the response variables were binary (male, not male; had calf, did not have calf), necessitating the use of a logit link function. The significance of the explanatory terms, the fixed effects, was assessed by their Wald statistics (distributed as χ²) for each term when fitted last in the model. All interaction terms were tested, but are not reported unless statistically significant. Analysis was performed in Genstat 5, version 3.2.

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There is evidence that physiological signals under healthy conditions may have a fractal temporal structure. Here we investigate the possibility that time series generated by certain physiological control systems may be members of a special class of complex processes, termed multifractal, which require a large number of exponents to characterize their scaling properties. We report evidence for a multifractal character in a biological dynamical system, the healthy human heartbeat, and show that the multifractal character and nonlinear properties of the healthy heart rate are encoded in the Fourier phases. We uncover a loss of multifractality for a life-threatening condition, congestive heart failure.

Biomedical signals are generated by complex self-regulating systems that process inputs with a broad range of characteristics. Many physiological time series, such as the one shown in Fig. 1a, are extremely inhomogeneous and non-stationary, fluctuating in an irregular and complex manner. The analysis of the fractal properties of such fluctuations has been restricted to second-order linear characteristics such as the power spectrum and the two-point autocorrelation function. These analyses reveal that the fractal behaviour of healthy, free-running physiological systems is often
characterized by 1/f-like scaling of the power spectra, $S(f)$, where $f$ is the frequency\textsuperscript{9–12}.

Monofractal signals are homogeneous, in the sense that they have the same scaling properties throughout the entire signal. Therefore, monofractal signals can be indexed by a single global exponent—the Hurst exponent $H$ (ref. 13). Multifractal signals, on the other hand, can be decomposed into many subsets characterized by different local Hurst exponents $h$, which quantify the local singular behaviour and thus relate to the local scaling of the time series. Thus, multifractal signals require many exponents to characterize their scaling properties fully\textsuperscript{14–16}.

The statistical properties of the different subsets characterized by these different exponents $h$ can be quantified by the function $D(h)$, where $D(h)$ is the fractal dimension of the subset of the time series characterized by the local Hurst exponent $h$. Thus, the multifractal approach for signals, a concept introduced in the context of multi-affine functions\textsuperscript{14,15}, has the potential to describe a wide class of signals that are more complex than those characterized by a single fractal dimension (such as classical 1/f noise).

We tested whether a large number of exponents is required to characterize heterogeneous heartbeat interval time series (Fig. 1) by undertaking multifractal analysis. The first problem is to extract the local value of $h$. To this end, we used methods derived from wavelet theory\textsuperscript{16}. The properties of the wavelet transform make wavelet methods attractive for the analysis of complex non-stationary time series such as those found in physiological systems\textsuperscript{17}. In particular, wavelets can remove polynomial trends that could cause box-counting techniques to fail to quantify the local scaling of the signal\textsuperscript{18}. Additionally, the time-frequency localization properties of wavelets makes them particularly useful for revealing the underlying hierarchy that governs the temporal distribution of the local Hurst exponents\textsuperscript{19}. Thus, the wavelet transform allows a reliable multifractal analysis to be performed\textsuperscript{18,19}. We used derivatives of the gaussian function as the analysing wavelet, which allowed us to estimate the singular behaviour and the corresponding exponent $h$ at a given location in the time series. The higher the order, $n$, of the derivative, the higher the order of the polynomial trends removed and the better the detection of the temporal structure of the local scaling exponents in the signal.

We evaluated the local exponent $h$ through the modulus of the maxima values of the wavelet transform at each point in the time series. We then estimated the scaling of the partition function $Z_q(a)$, which is defined as the sum of the $q$th powers of the local maxima of the modulus of the wavelet transform coefficients at scale $a$ (ref. 19).

For small scales, we expect

$$Z_q(a) = a^q \cdot \theta(q).$$  

For certain values of $q$, the exponents $\theta(q)$ have familiar meanings. In particular, $\theta(2)$ is related to the scaling exponent of the Fourier power spectra, $S(f) \sim f^{\beta}$, as $\beta = 2 + \theta(2)$. For positive $q$, $Z_q(a)$ reflects the scaling of the large fluctuations and strong singularities, whereas for negative $q$, $Z_q(a)$ reflects the scaling of the small fluctuations and weak singularities\textsuperscript{20}. Thus, the scaling exponents $\theta(q)$ can reveal different aspects of cardiac dynamics.

Monofractal signals display a linear $\theta(q)$ spectrum, $\theta(q) = qH - 1$, where $H$ is the global Hurst exponent. For multifractal signals, $\theta(q)$ is a nonlinear function: $\theta(q) = qH - D(h)$, where $h = d\theta/dq$ is not constant. The fractal dimension $D(h)$, introduced earlier, is related to $\theta(q)$ through a Legendre transform:

$$D(h) = qH - \theta(q).$$  

We analysed both daytime (12:00 to 18:00) and night-time (0:00 to 6:00) heartbeat time series records from healthy subjects, and the daytime records of patients with congestive heart failure. These data were obtained by Holter monitoring\textsuperscript{20}. Our database includes 18 healthy subjects (13 female and 5 male, with ages between 20 and 50, average 34.3 years), and 12 congestive heart failure subjects (3 female and 9 male, with ages between 22 and 71, average 60.8 years) in sinus rhythm (see Methods for details on data acquisition and preprocessing). For all subjects, for a broad range of positive and negative $q$, the partition function $Z_q(a)$ scales as a power law (Fig. 2a, b).

For all healthy subjects, $\theta(q)$ is a nonlinear function (Figs 2c and 3a), which indicates that the heart rate of healthy humans is a multifractal signal. Figure 3b shows that, for healthy subjects, $D(h)$ has non-zero values for a broad range of local Hurst exponents $h$. The multifractality of healthy heartbeat dynamics cannot be explained by activity, as we analyse data from subjects at night. Furthermore, this multifractal behaviour cannot be attributed to sleep-stage transitions, as we find multifractal features during daytime hours as well. The range of scaling exponents ($0 < h < 0.3$) with non-zero fractal dimension $D(h)$ indicates that the fluctuations in the healthy heartbeat dynamics exhibit anti-correlated behaviour ($h = 1/2$ corresponds to uncorrelated behaviour; $h > 1/2$ corresponds to correlated behaviour).

In contrast, heart-rate data from subjects with a pathological...
condition—congestive heart failure—show a clear loss of multifractality (Fig. 3a, b). For the heart-failure subjects, $\tau(q)$ is close to linear and $D(h)$ is nonzero only over a very narrow range of exponents $h$, indicating mono-fractal behaviour (Fig. 3).

Our results show that, for healthy subjects, local Hurst exponents in the range $0.07 < h < 0.17$ are associated with fractal dimensions close to one. This means that the subsets characterized by these local exponents are statistically dominant. On the other hand, for the heart-failure subjects, the statistically dominant exponents are confined to a narrow range of local Hurst exponents centred at $h = 0.22$. These results suggest that, for heart failure, the fluctuations are less anti-correlated than for healthy dynamics, as the dominant scaling exponents $h$ are closer to 1/2.

We systematically compared our method with other widely used methods of heart-rate time-series analysis\textsuperscript{21,22}. Several of these methods do not result in a fully consistent assignment of healthy

Figure 3  Multifractality in healthy dynamics. a. Multifractal spectrum $\tau(q)$ of the group averages for daytime and night-time records for 18 healthy subjects and 12 patients with congestive heart failure. The results show multifractal behaviour for the healthy group and different behaviour for the heart-failure group. b. Fractal dimensions $D(h)$ obtained through a Legendre transform from the group-averaged $\tau(q)$ spectra of a. The shape of $D(h)$ for the individual records and for the group average is broad, indicating multifractal behaviour. $D(h)$ for the heart-failure group is very narrow, indicating monofractality. The different form of $D(h)$ for the heart-failure group may reflect perturbation of the cardiac neuroautonomic control mechanisms associated with this pathology.
versus diseased subjects (see http://polymer.bu.edu/~amaral/Heart.html). Figure 4a shows the results of our method based on the multifractal formalism. Each subject’s dataset in the database is characterized by three quantities. The first quantity ($x$-axis) is the degree of multifractality, which is the difference between the maximum and minimum values of local Hurst exponent $h$ for each individual. The degree of multifractality is zero for a monofractal. The second quantity ($y$-axis) is the exponent value $\tau(q) = 3$ characterizing the scaling of the third moment, $Z_3(h)$. The third quantity ($z$-axis) is the standard deviation of the interbeat intervals. Blue spheres, healthy subjects; red spheres, heart-failure subjects. This multifractal approach discriminates healthy from heart-failure subjects.

We next blindly analysed a separate database containing 10 records, 5 from healthy individuals and 5 from patients with congestive heart failure. The time series in the new database are shorter than those in our database; on average they are only 2-h long (less than 8,000 beats). Figure 4b shows the projection on the $x$–$y$ plane of our data presented in Fig. 4a. The results for the blind test are shown in black. Our approach clearly separates the blind test subjects into two groups: 1, 3, 5, 6 and 10 fall in the healthy group, and 2, 4, 7, 8 and 9 in the heart-failure group. Unblinding the test code confirms these assignments. Thus, an analysis incorporating the multifractal method may add diagnostic power to contemporary analytic methods of heartbeat (and other physiological) time-series analysis.

The multifractality of heart-beat time series also enables us to quantify the greater complexity of the healthy dynamics compared to those of pathological conditions. Power-spectrum analysis defines the complexity of heart-beat dynamics through its scale-free behaviour, identifying a single scaling exponent as an index of healthy or pathological behaviour. Hence, the power spectrum cannot quantify the greater level of complexity of the healthy dynamics, reflected in the heterogeneity of the signal. In contrast, the multifractal analysis reveals this new level of complexity by the

![Image](https://example.com/image.png)
broad range of exponents necessary to characterize the healthy dynamics. Moreover, the change in shape of the $D(h)$ curve for the heart-failure group may provide insight into the changes in the cardiac control mechanisms resulting from this pathology.

To study the complexity of the healthy dynamics further, we performed two tests with surrogate time series. First, we generated a surrogate time series by shuffling the interbeat interval increments of a record from a healthy subject. The new signal preserves the distribution of interbeat interval increments but destroys the long-range correlations among them. Hence, the signal is a simple random walk, which is characterized by a single Hurst exponent $H = 1/2$ and exhibits monofractal behaviour (Fig. 5a). Second, we generated a surrogate time series by performing a Fourier transform on a record from a healthy subject, preserving the amplitudes of the Fourier transform but randomizing the phases, and then performing an inverse Fourier transform. This procedure eliminates non-linearities, preserving only the linear features of the original time series. The new surrogate signal has the same $1/f$ behaviour in the power spectrum as the original heart-beat time series; however, it exhibits monofractal behaviour (Fig. 5a). We repeated this test on a record from a heart-failure subject. In this case, there is a smaller change in the multifractal spectrum (Fig. 5b). The results suggest that the healthy heart-beat time series contains important phase correlations which are cancelled in the surrogate signal by the randomization of the Fourier phases, and that these correlations are weaker in heart-failure subjects. Furthermore, our analysis indicates that the observed multifractality is related to nonlinear features of the healthy heartbeat dynamics. Several studies have tested for nonlinear and deterministic properties in records of heart rates. 23–27. We have demonstrated an explicit relation between the nonlinear features (represented by the Fourier phase interactions) and the multifractality of healthy cardiac dynamics (Fig. 5).

From a physiological perspective, the detection of robust multifractal scaling in the heart-rate dynamics is of interest because it indicates that the control mechanisms regulating the heartbeat might interact as part of a coupled cascade of feedback loops in a system operating far from equilibrium. 18,28. Furthermore, these results indicate that the healthy heartbeat is even more complex than previously suspected, posing a challenge to ongoing efforts to develop realistic models of the control of heart rate and other processes under neuroautonomic regulation.

Methods
Heart failure (CHF) data were recorded using a Del Mar Avionics Model 445 Holter recorder and digitized at 250 Hz. Beats were labelled using ‘Aristotle’ arrhythmia analysis software, which labels each detected beat as normal, ventricular ectopic, supraventricular ectopic or unknown. The location of the R-wave peaks is determined with a resolution of 4 ms.

Healthy datasets were recorded using a Marquette Electronics series 8500 Holter recorder. Using a Marquette Electronics model 8000T Holter scanner, the tapes were then digitized at 128 Hz, scanned and annotated. The annotations were manually verified by an experienced Holter scanning technician. The location of the R-wave peaks was thus determined with a resolution of 8 ms.

The finite resolution implies that our estimates of the interbeat intervals are affected by a white noise due to estimation error. The signal-to-noise ratio for both healthy and heart failure is of the order of 100. Furthermore, the white noise due to the measurements would lead to the detection of a local Hurst exponent $h = 0$ at very small scales. For that reason, we have considered only scales larger than 16 beats.

From the beat annotation file, only the intervals (NN) between consecutive normal beats were determined; thus intervals containing non-normal beats were eliminated from the NN interval series. For the CHF data, an average of 2% (range, 0.1 to 0.6%) of the intervals were eliminated, and for the normal data an average of 0.01% (range 0 to 0.06%) were eliminated. No interpolation was done for eliminated intervals.

To eliminate outliers due to missed beat detections which would give rise to erroneously large intervals that may have been included in the NN interval series, a moving-window average filter was applied. For each set of five contiguous NN intervals, a local mean was computed, excluding the central interval. If the value of the central interval was greater than twice the local average, it was considered to be an outlier and excluded from the NN interval series. This criterion was applied to each NN interval in the series. For the CHF data, an average of 0.02% (range 0 to 0.1%) of the intervals were eliminated, and for the normal data an average of 0.07% (range 0 to 0.7%) were eliminated. No interpolation was done for eliminated intervals. Overall a total of 2% (range, 0.1 to 6%) of the total number of NN intervals were eliminated for the CHF data and 0.08% (range 0 to 0.7% max) for the normal data.

Next, we built a time series of increments between consecutive NN intervals and calculated their standard deviation. We then identified all pairs of adjacent increments with opposite signs and with an amplitude larger than 3 s.d. The values of the increments for each pair were replaced by linear interpolations of their values and the time series of NN interval time was reconstructed by integration from the filtered increments. About 1% of time intervals were corrected by this procedure.

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