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A stochastic model of river discharge fluctuations

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Abstract

We study the daily river flow fluctuations of 30 international rivers. Using the detrended fluctuation analysis, we study the correlations in the magnitudes of river flow increments (volatilities), and find power-law correlations in volatilities for time scales less than 1 year; these correlations almost disappear for time scales larger than 1 year. Using surrogate data test for nonlinearity, we show that correlations in the magnitudes of river flow fluctuations are a measure for nonlinearity. We propose a simple nonlinear stochastic model for river flow fluctuations that reproduces the main scaling properties of the river flow series as well as the correlations and periodicities in the magnitudes of river flow increments. According to our model, the source of nonlinearity observed in the data is an interaction between a long-term correlated process and the river discharge itself.

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1. Introduction

The climate system often exhibits irregular and complex behavior. Although the climate system is driven by the well-defined seasonal periodicity, it is also a subject to

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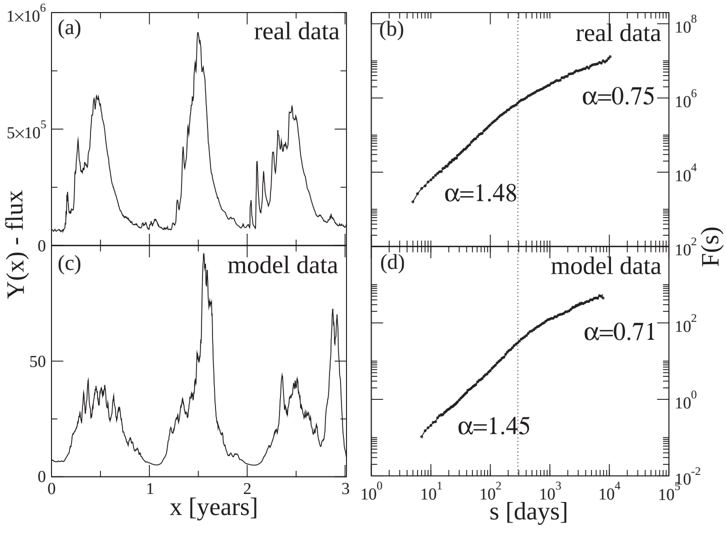

Fig. 1. (a) Flux series of a typical river (Columbia, 1986–1988); (b) DFA3 curve for Columbia river; (c) model data for three annual cycles; (d) DFA3 curve for a sample of model data.

unpredictable perturbations which can lead to extreme climate events. Here, we study one component of the climate system, the river discharge.

River flow can be characterized by several general features. As a result of the periodicity in precipitation, river flow has also strong seasonal periodicity. The seasonal cycle of river flow is asymmetric; i.e., river flow increases rapidly (usually during late winter and spring) and decreases gradually (toward the end of the autumn). The fluctuations in river flow are large for large river flow and small for small river flow. These features can be easily seen in the river flow data; see Fig. 1a. It is important to note that unlike other climate components, river flow may have a direct impact of human activity, like damming, use of river water for agriculture, etc., a fact which makes the river flow data more difficult to study.

The fluctuations in river flow are of special interest since they are directly linked to floods and droughts. There are several interesting characteristics of river flow fluctuations: (i) the river flow fluctuations have power law tails in the probability distribution [1,2], (ii) the river flow fluctuations are long-term correlated [3–5], and (iii) river flow fluctuations are multifractal [6–8]. These scaling laws may improve the statistical prediction of extreme changes in river flow [9].

Recently, we identified a new nonlinear aspect of river flow data [10]. The absolute values of river flow increments, the volatility, show a pronounced seasonal peak with several harmonics. However, after eliminating the nonlinearities of the river flow by randomizing the Fourier phases of the river flow increment series, the seasonal periodicity in the volatility series significantly weakened. Moreover, the volatility series exhibits long-term correlations which are destroyed after randomizing the Fourier phases of the river flow increment series. These volatility correlations are an additional indication for nonlinearity. These results suggest that absolute values of river flow increments tend to appear in clusters, in periodic and long-term correlated fashion.

Here, we study the correlation properties of river flow fluctuations, using the detrended fluctuations analysis (DFA) [11]. The DFA is capable of removing (polynomial) trends from the data—trends are known to have impact on the measuring of the scaling exponents that quantifies the long-term correlations in the data [12–14]. We present a simple stochastic model that reproduces several features of river flow fluctuations. This simple model may shed more light on the factors that attribute to the statistical properties of the river flow data.

2. Methodology

Some enumeration techniques like DFA, encounter difficulties when applied to periodic time series, like river flow records. We thus filtered out the periodicities of the river flow data before applying the DFA method. To exclude the seasonal trend and study the properties of the flux fluctuations only, we first differentiate the river flow time series $\widetilde{X}_j = X_{j+1} - X_j$, $j = 1, \ldots, N-1$ (N is the record length), then remove the seasonal cycle: $\Delta\widetilde{X}_j = \widetilde{X}_j - \langle\widetilde{X}_j\rangle$, where $\langle\widetilde{X}_j\rangle$ is the mean daily discharge (over the years of observation), and then divide by the seasonal standard deviation: $Z_j = \Delta\widetilde{X}_j / std(\Delta\widetilde{X}_j)$. We then integrate the series $x_k = \sum_{j=1}^{k} Z_j$, to compensate the first differentiation. Such a deseasoning of the increment series excludes seasonal periodicity of river flow data, as well as seasonal periodicity in the fluctuations around the seasonal average (i.e., it normalizes the large fluctuations around large river flow and small fluctuations around small river flow).

In recent years the DFA method has become a widely used tool for studying scaling properties of nonstationary time series [11]. It was applied successfully, e.g., to DNA sequences [15], heart-rate dynamics [16–18] and to econometric time series [19,20]. The n-order DFA consists of the following steps: (i) integrating the series under consideration, x_i, after subtracting the series average, $y(k) = \sum_{i=1}^{k} [x_i - \langle x\rangle]$, (ii) splitting the series y_k into segments of length s, (iii) calculating the root mean-square fluctuation function

$$F(s) \equiv \sqrt{\frac{1}{N}\sum_{k=1}^{N}[y(k) - y_s(k)]^2}\ ,$$

where $y_s(k)$ is the local trend and N is the series length. The local trend is evaluated by fitting a polynomial trend of order n in the corresponding segment. The nth order DFA detrends polynomial of order n from the profile $y(k)$. When the series follows a scaling law, we observe a power-law behavior for the fluctuation function

$$F(s) \sim s^{\alpha}\ ,$$

where α is called the scaling exponent. For uncorrelated records $\alpha = 0.5$, for correlated (persistent) records $\alpha > 0.5$, while for anti-correlated (anti-persistent) records $\alpha < 0.5$. Integration/differentiation of the series, will increase/reduce the exponent α by one.

In order to check the nonlinearity of river flow data, we use a surrogate data test for nonlinearity [21]. We define nonlinearity with respect to the Fourier phases—if the statistical properties of a time series solely depend on the power spectrum and the probability distribution regardless of the Fourier phases, the series is considered to be *linear*. Otherwise, the series is defined as *nonlinear*. For more details see Refs. [21,22]. In a surrogate data test the NULL hypothesis is that the series under consideration is linear. The surrogate data has the same probability distribution and almost the same power spectrum as the original series, but with random Fourier phases. If a statistical measure obtained for the original series is significantly different from that of the surrogate data, the NULL hypothesis is rejected and the series is considered to be nonlinear. We used the measures of periodic volatility and volatility correlations as measures for nonlinearity.

3. The model

To imitate the behavior of real hydrological time series, we consider artificial data with correlated noise on the background of (asymmetric) seasonal periodicity

$$y_{i+1} = (1-\gamma)y_i + Ay_i\eta_i + p_i \ ,$$

where y represents the river flow, γ is a damping coefficient, which bounds the fluctuations in river flow due to limited water sources, A is the fluctuation level, η is a long-term correlated noise, and p is an asymmetric periodic function

$$p_i = p_{j+nT} = \begin{cases} 1+\cos(2\pi f j) & \text{for } 0 \leqslant j < \frac{2}{3}T \ , \\ 1-\cos(4\pi f j) & \text{for } \frac{2}{3}T \leqslant j < T \ , \end{cases}$$

where T is the period ($T = 365$ days), j, n are integers, $0 \leqslant j < T$, $f = 3/4T$. When the noise level A increases, the nonlinear term $Ay_i\eta_i$ also increases. Such a model mimics the observed properties of river flow, i.e., the asymmetric periodic flow represented by p_i (see also [6] for observations of asymmetry), and the larger fluctuations for larger river flow represented by the product of the fluctuations η by the river flow level y. We used long-term correlated noise to mimic the persistence behavior found for time scales up to ~ 100 days. We tune the values of γ, the scaling exponent of correlated noise η, and the noise level A to fit the statistical properties of the river flow data; in our simulations we use $\gamma = 0.1$, $A = 0.04$, and correlated noise η with exponent $\alpha = 0.9$ and standard deviation 1. In Fig. 1c we present a typical example of the model's simulation. This series shows similar characteristics as seen in the data. We will show in the following that this simple scheme reproduces the linear and nonlinear properties of the data. We have generated 30 samples of model data and applied DFA and volatility analysis and compare the properties of the simulated series with those of real data.

4. Results

4.1. Scaling properties

We apply the DFA to hydrological time series of 30 rivers over the globe (daily records); the mean flux ranges from ~ 0.6 to $\sim 2 \times 10^5$ m^3/s and the series length ranges from 26 to 171 years with an average length of 81 years. We observed high correlations with an exponent (slope of power regression) $\alpha \sim 1.5$ for the period less than 100 days and a relatively smooth but clear change (crossover) to a smaller exponent $\alpha \sim 0.8$ for periods longer than $s \cong 100$ days (see Table 1). These results are in agreement with recent findings by Koscielny-Bunde et al. [5]. In Fig. 1b we show the DFA3 results of the representative Columbia river. The DFA curve exhibits crossover behavior: for small window size fluctuations are highly correlated with large scaling exponent while for larger window scales the correlations are weaker and characterized by a smaller scaling exponent. Model data exhibits similar behavior with similar scaling exponents. The DFA3 curve for the model data is presented in Fig. 1d. In Table 1 we compare the scaling exponents of the data with the scaling exponents of the model and find a good agreement between model's results and the data. We thus conclude that the model reproduces the scaling properties of the river flux data.

4.2. Volatility correlations

To apply the long-term volatility analysis [18,22], we consider the absolute values of river flux increments (seasonally filtered) time series, the volatility series, $\tilde{Y}_k = |Y_{k+1} - Y_k|$, $k = 1, \ldots, N-1$. We use the DFA3 to study the correlations in the volatility series. In the window range 1 week–1 year we obtain a correlation exponent $\alpha \sim 0.65$, and $\alpha \sim 0.5$ for window scales larger than 1 year (see results for the representative Columbia river in Fig. 2a). After applying the surrogate data test for nonlinearity, the exponent decreases to $\alpha \sim 0.5$ for window scales larger than one week (Fig. 2b).

Table 1
Results of DFA and volatility analysis for real data and model

Parameter	Range	Real data	Model data
DFA exponent	short	1.53 ± 0.24	1.58 ± 0.27
	long	0.87 ± 0.11	0.84 ± 0.13
Volatility exponent	short	0.67 ± 0.11	0.65 ± 0.13
	long	0.55 ± 0.12	0.51 ± 0.10

To obtain the mean ± 1 standard deviation we average the exponents of 30 world rivers and 30 realizations of the model (32K data points each). The exponents for the model are within the error bars of the data. The rivers with corresponding periods of observation are: Barron (79 y), Columbia (114 y), Danube (151 y), Divoka Orlice (83 y), Fraser (84 y), Gaula (90 y), Isar (39 y), Johnstone (74 y), Kinzig (82 y), Koher (111 y), Labe (102 y), Maas (80 y), Mary (76 y), Mitta (68 y), Murg (77 y), Naab (26 y), Niger (79 y), Orinoco (65 y), Regniz (30 y), Rhein (143 y), Severn (71 y), Severnaya Dvina (26 y), Susquehanna (96 y), Tana (51 y), Tauber (73 y), Thames (113 y), Vils (26 y), Wertach (77 y), Weser (171 y), Zaire (81 y).

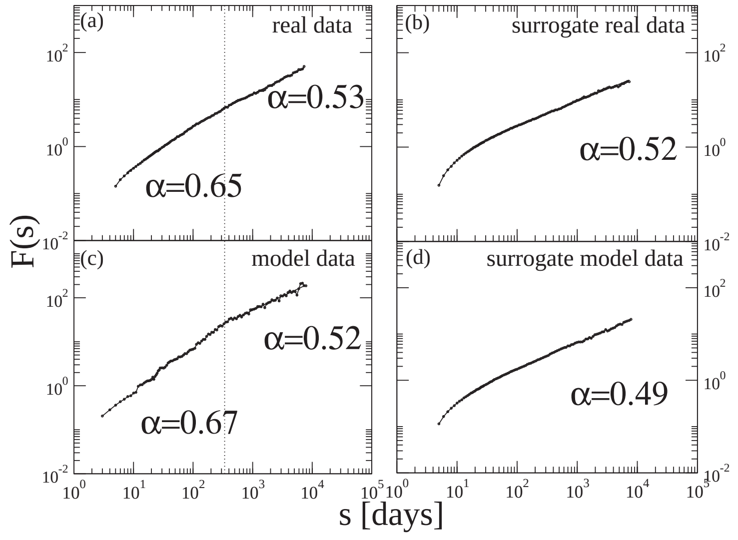

Fig. 2. Volatility: (a) DFA3 results for volatility series (Columbia river); (b) DFA results for phase-randomized volatility river series (compare to (a)); (c) DFA results for volatility series for a sample of model data; (d) DFA results for phase-randomized volatility model series (compare to (b)).

The results of surrogate data test, i.e., the decrease in the volatility exponent from large value to ~ 0.5, indicate the nonlinearity of the initial river flow time series. The same effect is observed in the simulated data (the results of the model and the surrogate data for the model are presented in Figs. 2c and 2d, respectively). The obtained value of the correlation exponent for surrogate model data, $\alpha \sim 0.49$, is close to the correspondent value for surrogate real data. A summary of the results for 30 world rivers and for 30 realizations of the model are given in Table 1.

4.3. Periodic volatility

We have shown [10] that power spectrum of volatility series have a pronounced seasonal peak, the periodic volatility, which disappears after the phase randomization procedure. This is again a sign of the presence of nonlinearity in the initial series. In Fig. 3 we show the power spectra of the absolute values of river flow increments (without applying the filtering procedure first), both for data and model, before and after applying the surrogate data test that randomizes the Fourier phases. The model shows similar spectra as for the data. Thus, the model data reproduces also the periodic volatility of the hydrological records.

5. Summary

We apply an advanced scaling technique, the DFA, to measure the correlation properties of river flow fluctuations. We find that the river flow fluctuations are highly

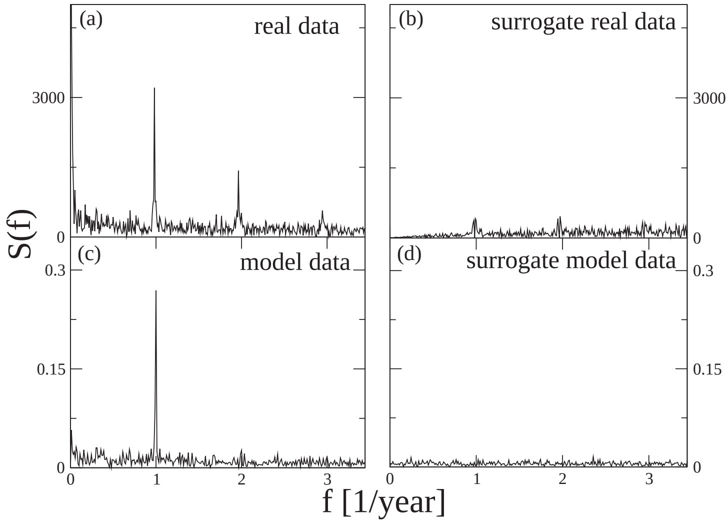

Fig. 3. Periodic volatility: (a) power spectrum for volatility series of real record (Columbia river); (b) results of phase randomization of series from (a); (c) power spectrum for volatility series of a sample of model data; (d) results of phase randomization of series from (b).

correlated for short time scales, up to 100 days ($\alpha \sim 1.5$), and less correlated for larger time scales ($\alpha \sim 0.8$). We also use the DFA to study correlation properties of the absolute values of river flow increments, the volatility. We find that the volatility series is correlated for time scales smaller than 1 year (with $\alpha \sim 0.65$) and that these correlations almost disappear for time scales larger than 1 year ($\alpha \sim 0.5$). By the use of surrogate data test, which randomizes the Fourier phases of the increment river flow series, we show that volatility correlations are an indication for nonlinearity [18,22], and that the nonlinearity decreases for time scales larger than 1 year.

We suggest a simple stochastic model for river flow that reproduces some statistical (linear and nonlinear) properties of the river flow data. The model exhibits asymmetric periodic behavior with large fluctuations around large river flow and small fluctuations around small river flow. The model reproduces the following properties of the river flow data: (i) the scaling exponents of the river flow series, including the crossover in the DFA curve, (ii) the scaling exponents of the volatility series for small and large time scales, and (iii) the periodic volatility of the volatility series. The results of our model suggest that the nonlinearity of the river flow reported in this study are due to interaction of river flow with correlated process.

Acknowledgements

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