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Improvements in the statistical approach to random Lévy flight searches

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Abstract

Recently it has been shown that the most efficient strategy for searching randomly located objects, when the sites are randomly distributed and can be revisited any number of times, leads to a power law distribution $P(\ell) = \ell^{-\mu}$ of the flights ℓ , with $\mu = 2$. We show analytically that the incorporation of energy considerations limits the possible range for the Lévy exponent μ , however, $\mu = 2$ still emerges as the optimal foraging condition. Furthermore, we show that the probability distribution of flight lengths for the short and intermediate flight length regimes depends on the details of the system. © 2001 Elsevier Science B.V. All rights reserved.

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The problem of searching randomly distributed sites whose exact locations are not known a priori has applications in fields ranging from information technology [1] to animal foraging [2–5]. For such problems, one of the most important questions we can ask is how to optimize the search of the target sites. It is not difficult to see that we generally need a mixing strategy: on the one hand the ignorance about the exact locations of the sites demands some sort of *probabilistic* approach to the problem. On the other hand, the search process itself requires specific rules of locomotion, leading then also to an *algorithmic* (one might even say “deterministic”) dynamical procedure.

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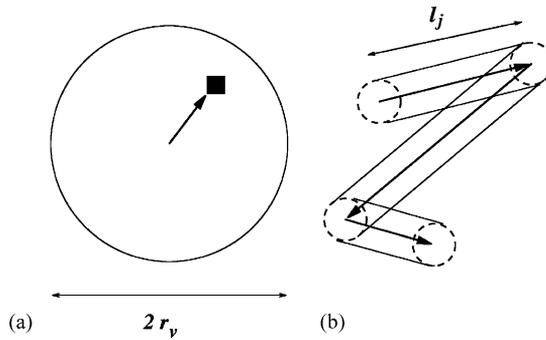


Fig. 1. Foraging strategy: (a) If there is a target site (full square) located within a “direct vision” distance r_v , then the forager detects it with certain probability and moves on a straight line to it. (b) If the forager does not detect a target site within a distance r_v , then the forager chooses a random direction and a random distance l_j from the Lévy probability distribution, and then proceeds as explained in the text.

In animal foraging, experimental data are obtained by making histograms of the flight lengths of the animals during their search for food. Such histograms [5–7] show that the foragers follow a Lévy probability distribution of long flight (or step) lengths ℓ_j . Lévy flights are characterized by a distribution function $P(\ell_j) \approx \ell_j^{-\mu}$, with $1 < \mu \leq 3$ (the Gaussian is the stable distribution for $\mu > 3$). The possible reason why this distribution is advantageous to the animals, relative to others, like the Gaussian or Poisson, are discussed, e.g., in [8,9]. A recent study [5] has addressed, in the context of animal foraging, the problem of identifying the most efficient strategy for food searching. It was found that when the target sites are sparse and can be visited any number of times, $\mu = 2$ emerges as the optimal parameter, a result which is consistent with experiments [8]. Conversely, if a target site can be visited only once, it was shown that the optimal value is theoretically $\mu \rightarrow 1$.

In the present contribution we shall go a step further and show how “dynamical” considerations can improve even more our statistical approach to Lévy flight random searches.

The starting point in Ref. [8] was to propose the following idealized model to describe foraging search. Assume that target sites are distributed randomly, and the forager behaves as follows (see Fig. 1):

(1) If there is a target site located within a ‘direct vision’ distance r_v , then the forager detects it and moves in a straight line towards the site.

(2) If there are no target sites within a distance r_v , then the forager chooses a direction at random and a distance l_j from the Lévy distribution. It then incrementally moves to the new point, constantly looking for a target site within a radius r_v along the way. If it does not detect a site, it stops after traversing the distance l_j . Then, the process starts all over again.

We now define the foraging efficiency, in a slightly different way from that in Ref. [8], as the ratio between the total energy gained in the visited sites to the total

distance traversed by the forager. Thus we have $\eta_E(\mu) = \langle E_s \rangle / \langle L_2 \rangle$, with $\langle L_2 \rangle$ the average distance traveled between two target sites, and $\langle E_s \rangle$ the mean net energy gained per site. We assume that $\langle E_s \rangle = g - r \langle L_2 \rangle$, where g is the mean energy gained per site and r is the rate at which energy is lost per unit length during the search between the two sites. Then $\eta_E(\mu) = g\eta(\mu) - r$, for $\eta = 1/\langle L_2 \rangle = 1/(\langle l \rangle N)$. Here $\langle l \rangle$ is the mean flight distance and N is the mean number of flights taken by a Lévy forager while traveling between two successive target sites. Due to the linear relation between $\eta_E(\mu)$ and $\eta(\mu)$, a value of μ which maximizes η also maximizes η_E . Since the η here is exactly the one in Ref. [8], all the results derived there are also valid for η_E which implies that $\mu=2$ is also the optimal value for the energy efficiency in the case of sparse food and when the animal can always return to the target sites.

The advantage here, however, is that now we have an energy constraint in the foraging process, i.e., we must impose $\langle E_s \rangle > 0$. Thus, the constraint, which depends on specific parameters of the system, for instance, g and r , becomes a fundamental factor for the forager to choose a foraging strategy. Indeed, if the biological parameters are such that $\langle E_s \rangle < 0$ when $\mu=2$ then the optimal foraging strategy can no longer be followed. An interesting example is given for amoeba by Schuster and Levandowsky [2–4], where experimental data show μ ranging from 2.0 to 2.5. In principle, the amoeba should follow a Lévy distribution with $\mu=2$. However, since the environment of these simple organisms can vary considerably (implying changed relevant parameters), one may conjecture that the different foraging strategies are changed from the optimal value because of energy constraints. Obviously, more experimental work is necessary to corroborate such a hypothesis.

We finally mention that depending on biological factors, more complicated expressions for $\langle E_s \rangle$ can emerge. Richer expressions for the efficiency, leading to different behaviors of η_E as a function of μ , can be used and thus force a change in the allowable range for the optimal value of μ . This point is under investigation and will be the subject of a future contribution [10].

A second point we want to discuss is related to short and intermediate flight length regimes. The two dynamical rules proposed above allow the animal to truncate its Lévy flight every time it finds food along its way. Since we are most interested in the limit of low target site (e.g., food) concentration C , then the number of flights which are aborted is smaller than the ones which are not. Thus, we still have a power law-like distribution of flight lengths but now with a small deviation from a pure Lévy case for the long flights. This is in fact observed experimentally as shown in Fig. 2. We treat this problem by considering that the search for food along a single flight is a Poisson process. So, we expect that the distribution of flight lengths would be modified to $P(l) = f(l)/l^\mu$, with $f(l)$ having an exponential behavior. Indeed, we show that [10] $f(l) = \mathcal{N}(\exp[-2r_v^2 C] - \exp[-2r_v l C])$ for $l > r_v$. Here \mathcal{N} is a normalization constant. To test our modified Lévy distribution we fit experimental data in Fig. 2 and it shows a very good agreement. From such fits a biologist could, for instance, estimate important parameters, like r_v and C . Our hope is that this “dynamical” approach would be helpful in the practical study of animal foraging.

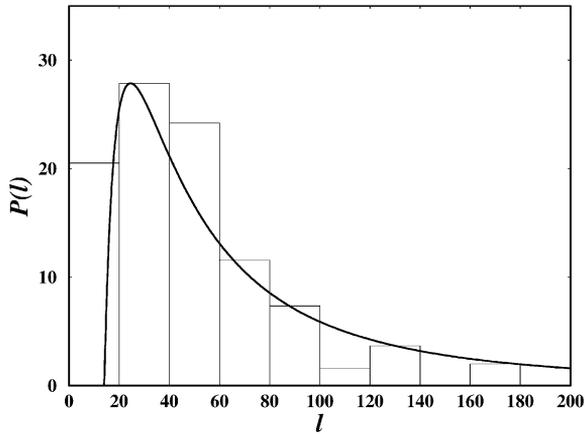


Fig. 2. Comparison of experimental data for deer (from Ref. [11]) with our modified flight distribution. The fitting parameters are $r_v = 14$ and $C = 0.001$. We see that our expression can fit well both long and short flight regimes. We mention that we also obtain good fits for bumble bees [10].

A general statistical approach for foraging leads to very general and robust results. Here we have shown that (for a particular model) the energy constraints on the system leave the maximum value of $\mu = 2$ for maximum efficiency unchanged, but this maximum is feasible only if the systems' parameters are such that the net energy gained during foraging is positive. Also, we have explicitly considered the Poisson process in searching for food along each flight chosen by the forager from a Lévy distribution. It leads to deviations from a pure Lévy distribution which can be measured experimentally. These calculations show that in principle one can determine important parameters for the system such as food concentration and direct vision.

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