Memory in the Occurrence of Earthquakes

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We study the statistics of the recurrence times τ between earthquakes above a certain magnitude M in six (one global and five regional) earthquake catalogs. We find that the distribution of the recurrence times strongly depends on the previous recurrence time τ_0, such that small and large recurrence times tend to cluster in time. This dependence on the past is reflected in both the conditional mean recurrence time and the conditional mean residual time until the next earthquake, which increase monotonically with τ_0. As a consequence, the risk of encountering the next event within a certain time span after the last event depends significantly on the past, an effect that has to be taken into account in any effective earthquake prognosis.

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Earthquakes, like many other natural hazards, are complex spatiotemporal phenomena, where the underlying mechanisms are not yet fully understood (for reviews, see [1–4]). Among the few well established basic laws are the Omori law [5] and the Gutenberg-Richter law [6]. The Omori law states that after a main earthquake the rate $n(t)$ of aftershocks above a certain magnitude M decays with time t as $n(t) \sim a_M t^{-\alpha}$ with α being independent of M and close to 1. The Gutenberg-Richter law describes the number $N(M)$ of earthquakes larger than M by $\log_{10} N(M) \approx -bM$, where b is close to 1. In addition, several modified frequency-magnitude (energy moment) distribution laws have been proposed, among them the generalized Pareto distribution for extreme earthquakes [7,8].

Recently, scaling laws for the temporal and spatial variability of the earthquakes have been obtained by Bak *et al.* [9] and Corral [10–12] which included various seismic regions with different tectonic properties. Considering the various tectonic environments as well as mainshocks and aftershocks as part of essentially one unique process, they analyzed the recurrence times between earthquakes greater than M in a large number of spatial areas of varying sizes $L \times L$. Bak *et al.* [9] concentrated on the distribution of the recurrence times in California and obtained a unified scaling law for the spatiotemporal set of data (see also [13,14]). Corral [10–12] studied the recurrence times in a large number of spatial areas of various sizes. He found the remarkable result that independent of the considered area and independent of the threshold M, the distribution $D_M(\tau)$ of recurrence times scales with the mean recurrence time $\bar{\tau}_M$ as

$$D_M(\tau) = \frac{1}{\bar{\tau}_M} f(\tau/\bar{\tau}_M), \tag{1}$$

where $f(\Theta)$ is a universal scaling function which does not depend on M and can be well approximated by the Gamma distribution

$$f(\Theta) \sim \Theta^{-(1-\gamma)} \exp(-\Theta^{\delta}/B), \tag{2}$$

with γ close to 0.6 and δ close to 1 [12].

Here we address the question of whether the distribution $D_M(\tau)$ [and thus $f(\Theta)$] fully characterizes the sequence of the return intervals. To this end, we study the conditional probability $D_M(\tau|\tau_0)$ defined as the distribution of those recurrence times τ that immediately follow a recurrence time τ_0. In records without memory, $D_M(\tau|\tau_0)$ does not depend on τ_0 and is identical to $D_M(\tau)$. Here we show that $D_M(\tau|\tau_0)$ depends strongly on the previous recurrence time τ_0, such that small recurrence times are more likely to be followed by small ones, and large recurrence times by large ones. This sequential clustering of earthquakes can be best observed in related quantities, like the conditional average of recurrence times $\hat{\tau}_M(\tau_0)$ and the mean conditional residual time to the next earthquake following τ_0, which both scale with $\bar{\tau}_M$. We show that the memory also influences significantly the conditional (risk) probability that after an event above M the next event will occur within time t, given that the previous event occurred time τ_0 before. Accordingly, the known information about the past can be used efficiently to improve the risk prognosis of earthquakes.

In our analysis, we consider six earthquake catalogs [one global world database Advanced National Seismic System (ANSS) and five regional catalogs (JUNEC, Kamchatka, NCSN, New Zealand, and SCEC)], for more details on these databases, see [15]. In each catalog, we are interested in the recurrence times τ between earthquakes with magnitudes above some threshold value M [see Fig. 1(a)]. Figure 1(b) shows a typical sequence of the recurrence times for the ANSS catalog for $M = 5$, whereas Fig. 1(c) shows the sequence of the recurrence times after shuffling the original sequence. The horizontal full lines in Figs. 1(b) and 1(c) represent the median recurrence time. In Fig. 1(b), one can see pronounced patches of small and large return times (above and below the median) that are clumped together. In contrast, in the shuffled magnitude sequence,

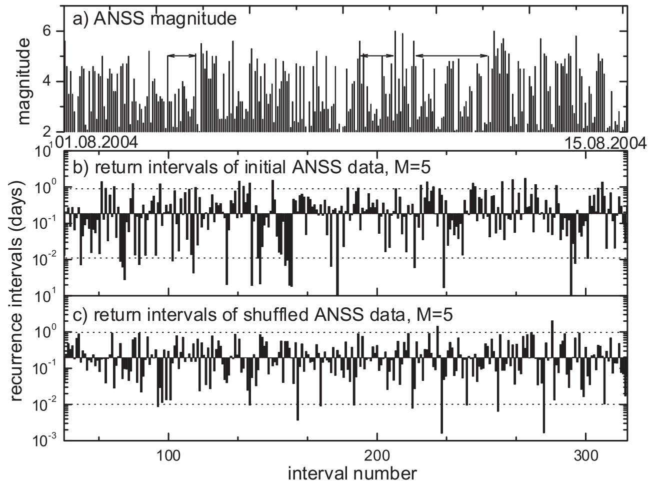

FIG. 1. (a) Sequence of earthquakes with magnitudes $M \geq 2$ for the global catalog of the Advanced National Seismic System. Three recurrence times for $M = 5$ are indicated by arrows. (b) Sequence of the recurrence times $\{\tau\}$ for the ANSS database above and below the median $\tau = 0.2d$. (c) Same as (b) but for shuffled magnitudes. The dotted lines are guides to the eye.

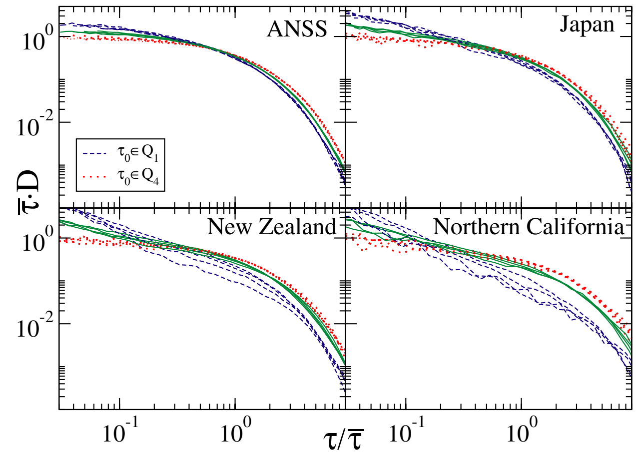

FIG. 2 (color online). Conditional probability distribution $D(\tau|\tau_0)$ for the recurrence times τ between earthquakes above the thresholds $M \simeq 2.5, 3, 3.5, 4$ that follow a recurrence time τ_0 either from the first quarter of the recurrence times (dashed blue lines) or from the last quarter (dotted red lines). The unconditional probability (solid green lines) is also shown. The results are for the global (ANSS) database and three regional databases (Japan University Network Earthquake Catalog, New Zealand GeoNet Project, and Northern California Seismic Network). To improve the statistics, we averaged over several threshold values in the neighborhood of each M, i.e., for $M \simeq 2.5$, we consider $2.3 < M < 2.7$, etc.

Fig. 1(c), such patches are almost absent. The patches in Fig. 1(b) demonstrate qualitatively the occurrence of a memory effect in the recurrence time sequence, where large (small) recurrence times tend to follow large (small) ones. To quantify these memory effects, we study the conditional distribution function $D_M(\tau|\tau_0)$. For simplicity, we drop the index M in D_M, $\bar{\tau}_M$ and all related quantities in the following. To obtain a reliable statistics, we do not determine $D(\tau|\tau_0)$ for a specific value of the precursor interval τ_0, but for a range of τ_0 values. To this end, we have sorted the full data set of N recurrence times in increasing order and divided it into four subrecords Q_1, Q_2, Q_3, and Q_4, such that each subrecord contains one quarter of the total number of recurrence times. Therefore, the $N/4$ smallest recurrence times are in Q_1, while the $N/4$ largest times are in Q_4. By choosing τ_0 this way, we actually keep $\tau_0/\bar{\tau}$ constant for different values of M. Figure 2 shows $\bar{\tau}D(\tau|\tau_0)$ for τ_0 in Q_1 (dashed lines) and Q_4 (dotted lines) as a function of $\tau/\bar{\tau}$, for M close to 2.5, 3, 3.5, and 4 in the global ANSS catalog as well as in the regional catalogs of Japan, New Zealand, and Northern California [16]. For comparison, we also show the respective unconditional distribution functions $D(\tau)$ (solid lines) [11].

As seen in Fig. 2, $D(\tau|\tau_0)$ depends strongly on the precursor interval τ_0 and changes its functional form according to τ_0. For τ_0 in Q_1, the probability of finding τ below (above) $\bar{\tau}$ is enhanced (decreased) compared with $D(\tau)$, while the opposite occurs for τ_0 in Q_4. It is interesting that the scaling with $\tau/\bar{\tau}$ is good not only for the unconditional distribution $D(\tau)$ [11], but also for the conditional probability $D(\tau|\tau_0)$ (provided the statistics are as good as in Figs. 2). Accordingly, to a good approximation, $D(\tau|\tau_0)$ can be written in the scaled form $D(\tau|\tau_0) = (1/\bar{\tau})g(\tau/\bar{\tau}, \tau_0/\bar{\tau})$. Because of this scaling, if $D(\tau|\tau_0)$ is known for one value of M, one can estimate it for other values of M, in particular, for large M (rare events) which are difficult to study due to a lack of statistics. Thus, knowledge of the past, i.e., τ_0 and the scaling function $g(\Theta, \Theta_0)$, improves the earthquake prognosis.

Closer inspection of Fig. 2 shows that for $\tau/\bar{\tau} < 1$, also the conditional distribution decays by a power law [see Eq. (2)], but with a history-dependent exponent γ. While for the unconditional distribution $\gamma \cong 0.6$ (in agreement with [12]), γ is modified to about 0.25 for τ_0 from Q_1 and about 0.85 for τ_0 from Q_4 (see Table I).

For further implications of the memory effect, we study the (conditional) mean recurrence time $\hat{\tau}(\tau_0)$ of those times that immediately follow a recurrence time τ_0. By definition, $\hat{\tau}(\tau_0)$ is the first moment of $D(\tau|\tau_0)$. From the scaling of $D(\tau|\tau_0)$ the analogous scaling $\hat{\tau}(\tau_0) = \bar{\tau}h(\tau_0/\bar{\tau})$ follows. To obtain $\hat{\tau}(\tau_0)$ with reliable statistics, we now divide the sorted (in increasing order) record of recurrence times into eight consecutive octaves, each one containing $N/8$ times. The first octave contains the shortest recurrence times, and so on. Now τ_0 represents the eight values of the mean recurrence time in the eight octaves. Figure 3 shows $\hat{\tau}(\tau_0)/\bar{\tau}$ as a function of $\tau_0/\bar{\tau}$. Because of the strong memory in the system, $\hat{\tau}/\bar{\tau}$ is well below one for $\tau_0/\bar{\tau}$ well below one, and well above one for $\tau_0/\bar{\tau}$ well above one. Accordingly, small and large recurrence times are more likely to be followed by small and large ones, respectively. Naturally, the effect of memory is more pronounced in the local catalogs than in the global one, where earthquakes

TABLE I. Values of the exponent γ of the scaling function $g(\Theta, \Theta_0)$ for the six databases studied, for Θ_0 (i) from the first quarter of intervals Q_1 and (ii) from the last quarter of intervals Q_4. To obtain γ, we assume that for a given Θ_0, g has the same scaling form as Eq. (2) (as suggested by Fig. 2). For the unconditional distribution, the γ values are consistent with results of Corral [11,12].

	NZ	ANSS	Japan	Kamchatka	NCSN	SCEC	Average
Q_1	0.3	0.5	0.35	0.25	0.1	0.1	0.27 ± 0.15
Q_4	0.8	0.9	0.85	0.95	0.8	0.75	0.84 ± 0.07
Uncond.	0.6	0.63	0.65	0.7	0.5	0.55	0.61 ± 0.07

from different areas are mixed. When the memory is destroyed by randomly shuffling the recurrence intervals, we obtain $\hat{\tau}(\tau_0)/\bar{\tau} \cong 1$, see Fig. 3, open symbols.

The conditional recurrence time $\hat{\tau}(\tau_0)$ tells us how long one has to wait, on average, for the next event to come, provided the two last events were separated by time τ_0. However, $\hat{\tau}(\tau_0)$ itself is not a good measure of the actual risk, since the standard deviation of the conditional recurrence times also increases with τ_0 and is of the same order of magnitude as $\hat{\tau}(\tau_0)$. To obtain a better estimation of the risk, we consider the conditional risk probability $P(t|\tau_0)$ (which also scales with $\bar{\tau}$) that following a recurrence time τ_0, another event (above the same threshold M) will occur within time t. Figure 4 shows representative results for $P(t|\tau_0)$ and $M = 4$ for τ_0 in the first and last octave of the recurrence intervals, for (a) the Northern California and (b) the New Zealand catalog. The curves reveal significant differences between the risk probability for small and the risk probability for large values of τ_0, which may be of considerable importance for risk estimation.

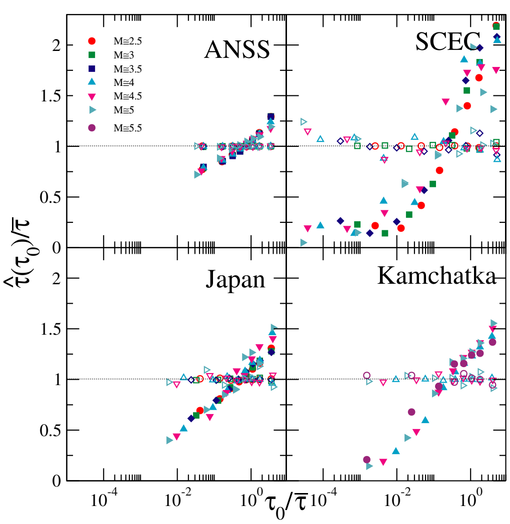

FIG. 3 (color online). Conditional recurrence time $\hat{\tau}(\tau_0)$ between earthquakes above certain threshold values M specified in the first panel, as a function of $\tau_0/\bar{\tau}$. The Kamchatka catalog only contains earthquakes with $M \geq 4$. As in Fig. 2, averages were taken over certain intervals around the threshold values to obtain better statistics. The horizontal lines represent $\hat{\tau}(\tau_0)/\bar{\tau} \simeq 1$, when there is no memory in the data. The open symbols are for the corresponding shuffled sequences of recurrence times.

Finally, we consider the residual time $\hat{\tau}(x|\tau_0)$ to the next event given that the time x has been elapsed since the last event and the last two events before were separated by the recurrence time τ_0. For $x = 0$, the expected residual time $\hat{\tau}(x = 0|\tau_0)$ is identical to $\hat{\tau}(\tau_0)$. In general, $\hat{\tau}(x|\tau_0)$ is related to $D(\tau|\tau_0)$ by

$$\hat{\tau}(x|\tau_0) = \int_x^\infty (\tau - x)D(\tau|\tau_0)d\tau \Big/ \int_x^\infty D(\tau|\tau_0)d\tau. \quad (3)$$

When the memory is irrelevant, $\hat{\tau}(x|\tau_0)$ does not depend on τ_0 and can be immediately obtained from $D(\tau)$. But since $D(\tau|\tau_0)$ depends strongly on τ_0 (see Fig. 2), we expect large memory effects also in the mean residual time. Moreover, since $D(\tau|\tau_0)$ deviates from a Poissonian, we expect that $\hat{\tau}(x|\tau_0)$ will increase with x [17], see also [12]. To reveal both dependencies in the seismic records, we focus on two values of x ($x = 0$ and $x = \bar{\tau}/2$) and two ranges of τ_0 values ("small" and "large" values referring to τ_0 below and above the median, respectively). We denote the average of $\hat{\tau}(x|\tau_0)$ over small τ_0 by $\hat{\tau}(x|\tau_0^-)$, and over large τ_0 by $\hat{\tau}(x|\tau_0^+)$. We compare both quantities with $\hat{\tau}(x)$, where overall τ_0 values have been averaged.

Figure 5 shows these three residual times for the six databases considered, for both $x = 0$ [Fig. 5(a)] and $x = \bar{\tau}/2$ [Fig. 5(b)]. As expected, for $x = \bar{\tau}/2$, the mean residual times to the next event are always larger than for $x = 0$. Because of the memory in the seismic activity, $\hat{\tau}(x|\tau_0^-)$ is significantly below $\hat{\tau}(x|\tau_0^+)$. The figure shows that the memory effect is most pronounced for the regional catalogs of Japan as well as for the two regional catalogs of

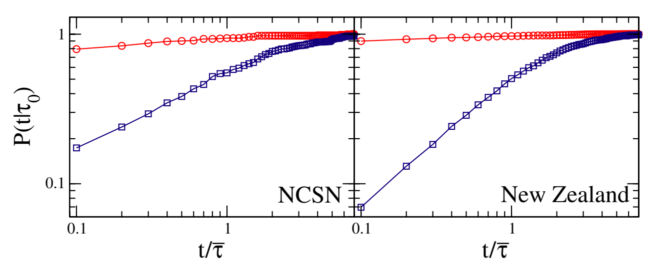

FIG. 4 (color online). Conditional probability $P(t|\tau_0)$ that two earthquakes (above a given threshold M), which are separated by the time τ_0, are followed by a third earthquake within time τ. The results are for the New Zealand GeoNet Project and the Northern California Seismic Network, with $M = 4$ and τ_0 being either in the first (circle) or last (square) octave.

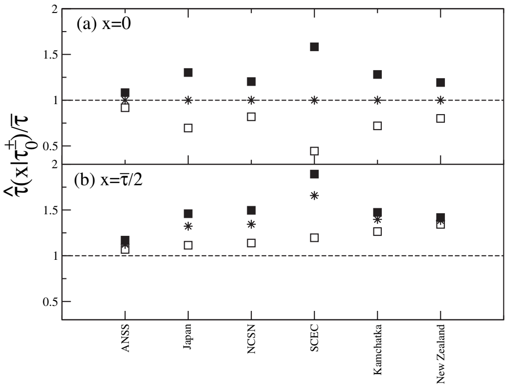

FIG. 5. Conditional residual times for the six databases considered, with $M \simeq 4.0$ for Kamchatka and $M \simeq 2.5$ for the other databases, for (a) $x = 0$ and (b) $x = \bar{\tau}/2$. In $\hat{\tau}(x|\tau_0^-)/\bar{\tau}$ and $\hat{\tau}(x|\tau_0^+)/\bar{\tau}$, averages have been performed over all precursor intervals τ_0 below (open symbols) and above (solid symbols) the median. To get a better statistics in (b), averages were taken over 20 equidistant x values between $0.4\bar{\tau}$ and $0.6\bar{\tau}$.

California. By definition, $\hat{\tau}(0) = (\hat{\tau}(0|\tau_0^-) + \hat{\tau}(0|\tau_0^+))/2$. This implies that $\hat{\tau}(0|\tau_0^-)$ and $\hat{\tau}(0|\tau_0^+)$ are symmetrical above and below $\hat{\tau}(0) \equiv \bar{\tau}$ as is also seen in Fig. 5(a). In contrast, due to the memory, this symmetry breaks for x above 0, since large elapsed times x are less frequent after short τ_0. Accordingly, the large τ_0 values (above the median) contribute more to the average value of $\hat{\tau}(x)$, which leads to the asymmetry seen in Fig. 5(b).

We would like to note that the memory found here should be distinguished from the memory occurring in the nonstationary regime after major earthquakes, where the rate of the aftershocks decreases in time by the Omori law. The decreasing rate generates a kind of memory where also small (large) recurrence intervals follow small (large) ones. To test if the memory we find is due to these aftershocks, we analyzed several time regimes in the catalogs (e.g., 1988–1991 and 1995–1998 in the SCEC catalog) where major events are absent, and found that the memory persists also in these regimes. This indicates that the aftershocks after major earthquakes cannot be the origin of our findings. However, we cannot exclude the possibility that the memory is due to other types of aftershocks that might occur after all scales of events, not only after the major ones, and are present in the whole catalog. If this is the case, our findings may be considered as a generalization of the Omori law.

In summary, we have studied the statistics of the recurrence times between earthquakes above certain threshold values M and observed a strong memory in the occurrence of these events such that small recurrence times are likely to be followed by small ones and large recurrence times by large ones. We have quantified this clustering of earthquakes by four quantities: (i) the conditional distribution function, (ii) the conditional mean recurrence time, (iii) the conditional risk function, and (iv) the conditional residual time; all of them differ significantly from the corresponding unconditional quantities. The memory effect may be particularly useful since (as seen in Fig. 4) it allows us to take into account the available relevant information from the past to obtain an improved risk estimation. Similar memory effects, due to long-term persistence, have been found recently in climate [18].

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[16] To improve the statistics of the histogram, we use logarithmic binning. We consider recurrence times between 2 min and $10\bar{\tau}$, and divide this range into 50 log bins. In each of these logarithmically equidistant bins, we count the number of recurrence times and divide it by the size of the bin.
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