Memory in the Occurrence of Earthquakes

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We study the statistics of the recurrence times \( \tau \) between earthquakes above a certain magnitude \( M \) in six (one global and five regional) earthquake catalogs. We find that the distribution of the recurrence times strongly depends on the previous recurrence time \( \tau_0 \), such that small and large recurrence times tend to cluster in time. This dependence on the past is reflected in both the conditional mean recurrence time and the conditional mean residual time until the next earthquake, which increase monotonically with \( \tau_0 \). As a consequence, the risk of encountering the next event within a certain time span after the last event depends significantly on the past, an effect that has to be taken into account in any effective earthquake prognosis.

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Earthquakes, like many other natural hazards, are complex spatiotemporal phenomena, where the underlying mechanisms are not yet fully understood (for reviews, see [1–4]). Among the few well established basic laws are the Omori law [5] and the Gutenberg-Richter law [6]. The Omori law states that after a main earthquake the rate \( n(t) \) of aftershocks above a certain magnitude \( M \) decays with time \( t \) as \( n(t) \sim a_M t^{-\alpha} \) with \( \alpha \) being independent of \( M \) and close to 1. The Gutenberg-Richter law describes the number \( N(M) \) of earthquakes larger than \( M \) by \( \log_{10} N(M) = -bM \), where \( b \) is close to 1. In addition, several modified frequency-magnitude (energy moment) distribution laws have been proposed, among them the generalized Pareto distribution for extreme earthquakes [7,8].

Recently, scaling laws for the temporal and spatial variability of the earthquakes have been obtained by Bak et al. [9] and Corral [10–12] which included various seismic regions with different tectonic properties. Considering the various tectonic environments as well as mainshocks and aftershocks as part of essentially one unique process, they analyzed the recurrence times between earthquakes greater than \( M \) in a large number of spatial areas of varying sizes \( L \times L \). Bak et al. [9] concentrated on the distribution of the recurrence times in California and obtained a unified scaling law for the spatiotemporal set of data (see also [13,14]). Corral [10–12] studied the recurrence times in a large number of spatial areas of various sizes. He found the remarkable result that independent of the considered area and independent of the threshold \( M \), the distribution \( D_M(\tau) \) of recurrence times scales with the mean recurrence time \( \bar{\tau}_M \) as

\[
D_M(\tau) = \frac{1}{\bar{\tau}_M} f(\tau/\bar{\tau}_M),
\]

where \( f(\Theta) \) is a universal scaling function which does not depend on \( M \) and can be well approximated by the Gamma distribution

\[
f(\Theta) \sim \Theta^{-(1-\gamma)} \exp(-\Theta^{\delta}/B),
\]

with \( \gamma \) close to 0.6 and \( \delta \) close to 1 [12].

Here we address the question of whether the distribution \( D_M(\tau) \) [and thus \( f(\Theta) \)] fully characterizes the sequence of the return intervals. To this end, we study the conditional probability \( D_M(\tau|\tau_0) \) defined as the distribution of those recurrence times \( \tau \) that immediately follow a recurrence time \( \tau_0 \). In records without memory, \( D_M(\tau|\tau_0) \) does not depend on \( \tau_0 \) and is identical to \( D_M(\tau) \). Here we show that \( D_M(\tau|\tau_0) \) depends strongly on the previous recurrence time \( \tau_0 \), such that small recurrence times are more likely to be followed by small ones, and large recurrence times by large ones. This sequential clustering of earthquakes can be best observed in related quantities, like the conditional average of recurrence times \( \bar{\tau}_M(\tau_0) \) and the mean conditional residual time to the next earthquake following \( \tau_0 \), which both scale with \( \bar{\tau}_M \). We show that the memory also influences significantly the conditional (risk) probability that after an event above \( M \) the next event will occur within time \( \tau \), given that the previous event occurred time \( \tau_0 \) before. Accordingly, the known information about the past can be used efficiently to improve the risk prognosis of earthquakes.

In our analysis, we consider six earthquake catalogs [one global world database Advanced National Seismic System (ANSS) and five regional catalogs (JUNEC, Kamchatka, NCSN, New Zealand, and SCEC)], for more details on these databases, see [15]. In each catalog, we are interested in the recurrence times \( \tau \) between earthquakes with magnitudes above some threshold value \( M \) [see Fig. 1(a)]. Figure 1(b) shows a typical sequence of the recurrence times for the ANSS catalog for \( M = 5 \), whereas Fig. 1(c) shows the sequence of the recurrence times after shuffling the original sequence. The horizontal full lines in Figs. 1(b) and 1(c) represent the median recurrence time. In Fig. 1(b), one can see pronounced patches of small and large return times (above and below the median) that are clumped together. In contrast, in the shuffled magnitude sequence,
FIG. 1. (a) Sequence of earthquakes with magnitudes $M \geq 2$ for the global catalog of the Advanced National Seismic System. Three recurrence times for $M = 5$ are indicated by arrows. (b) Sequence of the recurrence times $[\tau]$ for the ANSS database above and below the median $\tau = 0.2d$. (c) Same as (b) but for shuffled magnitudes. The dotted lines are guides to the eye.

As seen in Fig. 2, $D(\tau|\tau_0)$ depends strongly on the precursor interval $\tau_0$ and changes its functional form according to $\tau_0$. For $\tau_0$ in $Q_1$, the probability of finding $\tau$ below (above) $\bar{\tau}$ is enhanced (decreased) compared with $D(\tau)$, while the opposite occurs for $\tau_0$ in $Q_1$. It is interesting that the scaling with $\tau/\bar{\tau}$ is good not only for the unconditional distribution $D(\tau)$ [11], but also for the conditional probability $D(\tau|\tau_0)$ (provided the statistics are as good as in Figs. 2). Accordingly, to a good approximation, $D(\tau|\tau_0)$ can be written in the scaled form $D(\tau|\tau_0) = (1/\bar{\tau})g(\tau/\bar{\tau}, \tau_0/\bar{\tau})$. Because of this scaling, if $D(\tau|\tau_0)$ is known for one value of $M$, one can estimate it for other values of $M$, in particular, for large $M$ (rare events) which are difficult to study due to a lack of statistics. Thus, knowledge of the past, i.e., $\tau_0$ and the scaling function $g(\Theta, \Theta_0)$, improves the earthquake prognosis.

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from different areas are mixed. When the memory is destroyed by randomly shuffling the recurrence intervals, we obtain \( \hat{\tau}(\tau_0)/\bar{\tau} \equiv 1 \), see Fig. 3, open symbols.

The conditional recurrence time \( \hat{\tau}(\tau_0) \) tells us how long one has to wait, on average, for the next event to come, provided the two last events were separated by time \( \tau_0 \). However, \( \hat{\tau}(\tau_0) \) itself is not a good measure of the actual risk, since the standard deviation of the conditional recurrence times also increases with \( \tau_0 \) and is of the same order of magnitude as \( \bar{\tau}(\tau_0) \). To obtain a better estimation of the risk, we consider the conditional risk probability \( P(t|\tau_0) \) (which also scales with \( \bar{\tau} \)) that following a recurrence time \( \tau_0 \), another event (above the same threshold \( M \)) will occur within time \( t \). Figure 4 shows representative results for \( P(t|\tau_0) \) and \( M = 4 \) for \( \tau_0 \) in the first and last octave of the recurrence intervals, for (a) the Northern California and (b) the New Zealand catalog. The curves reveal significant differences between the risk probability for small and the risk probability for large values of \( \tau_0 \), which may be of considerable importance for risk estimation.

Table I. Values of the exponent \( \gamma \) of the scaling function \( g(\Theta, \Theta_0) \) for the six databases studied, for \( \Theta_0 \) (i) from the first quarter of intervals \( Q_1 \) and (ii) from the last quarter of intervals \( Q_4 \). To obtain \( \gamma \), we assume that for a given \( \Theta_0 \), \( g \) has the same scaling form as Eq. (2) (as suggested by Fig. 2). For the unconditional distribution, the \( \gamma \) values are consistent with results of Corral [11,12].

<table>
<thead>
<tr>
<th></th>
<th>NZ</th>
<th>ANSS</th>
<th>Japan</th>
<th>Kamchatka</th>
<th>NCSN</th>
<th>SCEC</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_1 )</td>
<td>0.3</td>
<td>0.5</td>
<td>0.35</td>
<td>0.25</td>
<td>0.1</td>
<td>0.1</td>
<td>0.27 ± 0.15</td>
</tr>
<tr>
<td>( Q_4 )</td>
<td>0.8</td>
<td>0.9</td>
<td>0.85</td>
<td>0.95</td>
<td>0.8</td>
<td>0.75</td>
<td>0.84 ± 0.07</td>
</tr>
<tr>
<td>Uncond.</td>
<td>0.6</td>
<td>0.63</td>
<td>0.65</td>
<td>0.7</td>
<td>0.5</td>
<td>0.55</td>
<td>0.61 ± 0.07</td>
</tr>
</tbody>
</table>

Finally, we consider the residual time \( \hat{\tau}(x|\tau_0) \) to the next event given that the time \( x \) has been elapsed since the last event and the last two events before were separated by the recurrence time \( \tau_0 \). For \( x = 0 \), the expected residual time \( \hat{\tau}(x = 0|\tau_0) \) is identical to \( \hat{\tau}(\tau_0) \). In general, \( \hat{\tau}(x|\tau_0) \) is related to \( D(\tau|\tau_0) \) by

\[
\hat{\tau}(x|\tau_0) = \int_x^\infty (\tau - x)D(\tau|\tau_0)d\tau/\int_x^\infty D(\tau|\tau_0)d\tau. \tag{3}
\]

When the memory is irrelevant, \( \hat{\tau}(x|\tau_0) \) does not depend on \( \tau_0 \) and can be immediately obtained from \( D(\tau) \). But since \( D(\tau|\tau_0) \) depends strongly on \( \tau_0 \) (see Fig. 2), we expect large memory effects also in the mean residual time. Moreover, since \( D(\tau|\tau_0) \) deviates from a Poissonian, we expect that \( \hat{\tau}(x|\tau_0) \) will increase with \( x \) [17], see also [12]. To reveal both dependencies in the seismic records, we focus on two values of \( x \) (\( x = 0 \) and \( x = \bar{\tau}/2 \)) and two ranges of \( \tau_0 \) values ("small" and "large" values referring to \( \tau_0 \) below and above the median, respectively). We denote the average of \( \hat{\tau}(x|\tau_0) \) over small \( \tau_0 \) by \( \hat{\tau}(x|\tau_0) \), and over large \( \tau_0 \) by \( \hat{\tau}(x|\tau_0) \). We compare both quantities with \( \hat{\tau}(x) \), where overall \( \tau_0 \) values have been averaged.

Figure 5 shows these three residual times for the six databases considered, for both \( x = 0 \) [Fig. 5(a)] and \( x = \bar{\tau}/2 \) [Fig. 5(b)]. As expected, for \( x = \bar{\tau}/2 \), the mean residual times to the next event are always larger than for \( x = 0 \). Because of the memory in the seismic activity, \( \hat{\tau}(x|\tau_0) \) is significantly below \( \hat{\tau}(x|\tau_0) \). The figure shows that the memory effect is most pronounced for the regional catalogs of Japan as well as for the two regional catalogs of

![Figure 3](image-url)  
*Fig. 3 (color online).* Conditional recurrence time \( \hat{\tau}(\tau_0) \) between earthquakes above certain threshold values \( M \) specified in the first panel, as a function of \( \tau_0/\bar{\tau} \). The Kamchatka catalog only contains earthquakes with \( M \geq 4 \). As in Fig. 2, averages were taken over certain intervals around the threshold values to obtain better statistics. The horizontal lines represent \( \hat{\tau}(\tau_0)/\bar{\tau} \approx 1 \), when there is no memory in the data. The open symbols are for the corresponding shuffled sequences of recurrence times.

![Figure 4](image-url)  
*Fig. 4 (color online).* Conditional probability \( P(t|\tau_0) \) that two earthquakes (above a given threshold \( M \)) are separated by a time \( \tau_0 \), are followed by a third earthquake within time \( \tau \). The results are for the New Zealand GeoNet Project and the Northern California Seismic Network, with \( M = 4 \) and \( \tau_0 \) being either in the first (circle) or last (square) octave.
time; all of them differ significantly from the corresponding unconditional quantities. The memory effect may be particularly useful since (as seen in Fig. 4) it allows us to take into account the available relevant information from the past to obtain an improved risk estimation. Similar memory effects, due to long-term persistence, have been found recently in climate [18].

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[16] To improve the statistics of the histogram, we use logarithmic binning. We consider recurrence times between 2 min and 109, and divide this range into 50 log bins. In each of these logarithmically equidistant bins, we count the number of recurrence times and divide it by the size of the bin.