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# Recurrence intervals between earthquakes strongly depend on history

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## Abstract

We study the statistics of the recurrence times between earthquakes above a certain magnitude  $M$  in California. We find that the distribution of the recurrence times strongly depends on the previous recurrence time  $\tau_0$ . As a consequence, the conditional mean recurrence time  $\hat{\tau}(\tau_0)$  between two events increases monotonically with  $\tau_0$ . For  $\tau_0$  well below the average recurrence time  $\bar{\tau}$ ,  $\hat{\tau}(\tau_0)$  is smaller than  $\bar{\tau}$ , while for  $\tau_0 > \bar{\tau}$ ,  $\hat{\tau}(\tau_0)$  is greater than  $\bar{\tau}$ . Also, the mean residual time until the next earthquake does not depend only on the elapsed time, but also strongly on  $\tau_0$ . The larger  $\tau_0$  is, the larger is the mean residual time. The above features should be taken into account in any earthquake prognosis.

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Recently, Corral [1,2] studied the recurrence of earthquakes above a certain magnitude threshold  $M$  in spatial areas delimited by a window of  $L$  degrees in longitude and  $L$  degrees in latitude. He found that the distribution  $D(\tau)$  of recurrence

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times  $\tau$  scales with the mean recurrence time  $\bar{\tau}$  as

$$D(\tau) = \frac{1}{\bar{\tau}} f(\tau/\bar{\tau}),$$

where the function  $f(\Theta)$  is quite universal and independent of  $M$ . For  $\Theta$  below 1,  $f$  can be approximated by a power-law, while for  $\Theta \gg 1$ ,  $f(\Theta)$  decays exponentially with  $\Theta$ . As a consequence of the deviation from a Poissonian decay [3], the mean residual time to the next event increases with the elapsed time [1,2].

In this paper, we study the statistics of the recurrence intervals of the California database [4] and find that both quantities, the recurrence interval distribution  $D(\tau)$  and the mean residual time to the next earthquake strongly depend on the previous recurrence time interval  $\tau_0$ .

We study the records from the local Californian earthquake catalog [4] for the period 1981–2003 in the area 30.5–38.5° N latitude, 114–122° W longitude, with minimal magnitude threshold value  $M = 2$  and minimal recurrence times 2 min [5]. Similarly to Corral [1,6], we consider the earthquakes in the region that are above a certain threshold  $M$ , as a linear process in time  $\{t_i\}$  without taking into account the spatial coordinates of the event hypocenters. We are interested in the recurrence intervals  $\tau_i = t_i - t_{i-1}$  between these earthquakes.

In records without memory, the (conditional) distribution  $D(\tau|\tau_0)$  of recurrence intervals  $\tau$  that directly follow a certain interval  $\tau_0$ , does not depend on the value of  $\tau_0$  and is identical to  $D(\tau)$ . In contrast, in records with long-term memory, there is a pronounced dependence of  $D(\tau|\tau_0)$  on  $\tau_0$  [7,8]. To study possible memory effects in the earthquake records with a reliable statistics, we have studied the conditional distribution  $D(\tau|\tau_0)$  not for a specific  $\tau_0$  value, but for values of  $\tau_0$  in certain intervals. To this end, we have sorted the record of  $N$  recurrence intervals in increasing order and divided it into four subrecords  $Q_1$ ,  $Q_2$ ,  $Q_3$  and  $Q_4$ , such that each subrecord contains one quarter of the total number of recurrence intervals. By definition, the  $N/4$  lowest recurrence intervals are in  $Q_1$ , while the  $N/4$  largest intervals are in  $Q_4$ .

Fig. 1 shows  $D(\tau|\tau_0)$  for  $\tau_0$  averaged over  $Q_1$  and  $Q_4$ . For comparison, we also show the unconditional distribution function  $D(\tau)$ . To improve the statistics, we used logarithmic binning. We considered time scales from 2 min to  $10\bar{\tau}$ , with 50 log-bins, counted the number of recurrence intervals within each bin and divided it by the size of the bin. To further improve the statistics, we averaged the probability distribution over threshold values  $M = 2.25 \dots 2.75$  around  $M \simeq 2.5$ . Finally, we normalized the probability distribution to obtain the probability densities of interest. The figure shows that for  $\tau$  well below its mean value  $\bar{\tau}$ , the probability of finding  $\tau$  below (above)  $\bar{\tau}$  is enhanced (decreased) compared with  $D(\tau)$  for  $\tau_0$  in  $Q_1$ , while the opposite occurs for  $\tau_0$  in  $Q_4$ .

By definition,  $\hat{\tau}(\tau_0)$  is the mean recurrence intervals, when the two events before were separated by an interval  $\tau_0$ . The memory effect in the conditional distribution function  $D(\tau|\tau_0)$  leads to an explicit dependence of  $\hat{\tau}(\tau_0)$  on  $\tau_0$ . To calculate  $\hat{\tau}(\tau_0)$ , we divided the sorted (in increasing order) record of recurrence intervals into 8 consecutive octaves. Each octave contains  $N/8$  intervals. In each interval, we calculate the mean value. We studied  $\hat{\tau}$  as a function of  $\tau_0/\bar{\tau}$ , where now  $\tau_0$  denotes

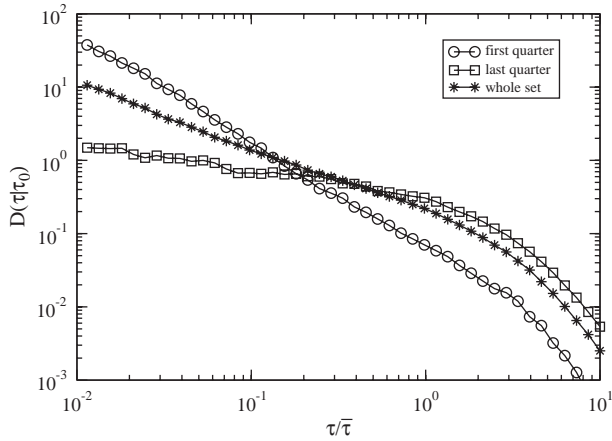


Fig. 1. Conditional probability distribution for the recurrence time intervals between earthquakes above a threshold  $M \simeq 2.5$  following recurrence time  $\tau_0$  from the first quarter (circles) and the last quarter (squares) of the recurrence time, and the unconditional probability (stars). To improve statistics, averages were taken for  $2.25 \leq M \leq 2.75$ .

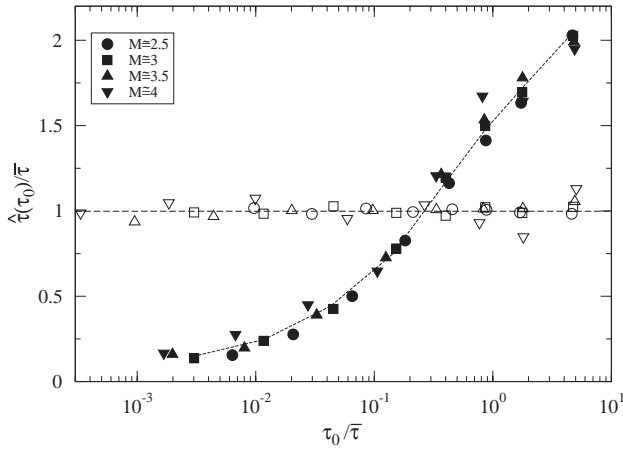


Fig. 2. Expected recurrence time  $\hat{\tau}(\tau_0)$  between earthquakes above thresholds  $M \simeq 2.5$  (full circles),  $M \simeq 3$  (full squares),  $M \simeq 3.5$  (full triangles up), and  $M \simeq 4$  (full triangle down) following  $\tau_0$  taken from the eight octaves described in the text. Averages are taken in intervals  $M \pm 0.25$  to obtain better statistics. The open symbols represent the analysis of the randomly shuffled recurrence time record, yielding  $\hat{\tau}(\tau_0)/\bar{\tau} \simeq 1$ .

the mean recurrence time in the octave. Fig. 2 shows  $\hat{\tau}(\tau_0)/\bar{\tau}$  as a function of  $\tau_0/\bar{\tau}$  and clearly demonstrates the strong effect of the memory. Small and large recurrence intervals are more likely to be followed by small and large ones, respectively,  $\hat{\tau}/\bar{\tau}$  is well below (above) one for  $\tau_0/\bar{\tau}$  well below (above) one. When the recurrence

intervals are randomly shuffled (no memory), we obtain  $\hat{\tau}(\tau_0)/\bar{\tau} \cong 1$ , see Fig. 2, open symbols.

A more general quantity is the expected residual time  $\hat{\tau}(x|\tau_0)$  to the next event, when time  $x$  has been already elapsed. For  $x = 0$ ,  $\hat{\tau}(0|\tau_0)$  is identical to  $\hat{\tau}(\tau_0)$ . In general,  $\hat{\tau}(x|\tau_0)$  is related to  $D(\tau|\tau_0)$  by

$$\hat{\tau}(x|\tau_0) = \int_x^\infty (\tau - x)D(\tau|\tau_0) d\tau / \int_x^\infty D(\tau|\tau_0) d\tau. \tag{1}$$

For uncorrelated records,  $D(\tau|\tau_0)$  is Poissonian, and  $\hat{\tau}(x|\tau_0)/\bar{\tau} = 1$ .

Fig. 3 clearly shows that  $\hat{\tau}(x|\tau_0)$  depends on both  $x$  and  $\tau_0$ . With increasing  $x$ , the expected residual time to the next event increases, as is shown in Fig. 3, for values of  $\tau_0$  from  $Q_1$  and  $Q_4$  (top and bottom curves). Thus, when  $\tau_0$  increases,  $\hat{\tau}(x|\tau_0)$  increases for all values of  $x$ . The middle curve shows the expected residual time averaged over all  $\tau_0$ , i.e., the unconditional residual time  $\hat{\tau}(x)$ . In this case, the interval between the last two events is not taken into account, and the slower-than-Poisson-decrease of the unconditional distribution function  $D(\tau)$ , Eq. (1), leads to the anomalous increase of the mean residual time with the elapsed time [3].

Our results for the unconditional residual time function for Californian earthquakes are very similar to the results of Corral [6] that were obtained for worldwide earthquake records. As shown here, there is a strong memory in the earthquake recurrence intervals, which influences significantly the residual time. Similar memory effects have been obtained recently for river flux, temperature and precipitation records (see Refs. [7,8]).

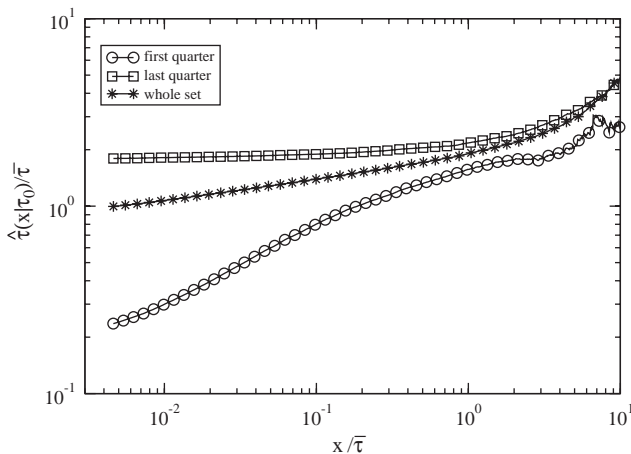


Fig. 3. Conditional mean residual time to the next earthquake above a threshold  $M \simeq 2.5$  following recurrence time  $\tau_0$  taken from the first quarter (bottom curve) and the last quarter (top curve) of the recurrence intervals, and unconditional mean residual time (middle curve). To improve statistics, average is taken for  $2.25 \leq M \leq 2.75$ .

To summarize, we have studied the memory effect in the earthquake events and showed that the distribution of the recurrence times and the mean residual time until the next earthquake strongly depend on the previous recurrence time. The conditional mean recurrence time between two events monotonically increases with  $\tau_0$ . These results should be taken into account in an efficient risk evaluation and forecasting of earthquakes. It is very plausible that the origin of these effects is due to long-term persistence in the earthquake occurrence.

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