Phase-Synchronization Decay of Fixational Eye Movements

SHAY MOSHEL, a JINRONG LIANG, a,b AVI CASPI, c RALF ENGBERT, d REINHOLD KLEIGL, d SHLOMO HAVLIN, a AND ARI Z. ZIVOTOFSKY c

aDepartment of Physics, Bar-Ilan University, Ramat-Gan, Israel
bDepartment of Mathematics, East China Normal University, Shanghai, China
cGonda Brain Research Center, Bar-Ilan University, Ramat-Gan, Israel
dDepartment of Psychology, University of Potsdam, Germany

ABSTRACT: In nonstationary noisy systems the traditional cross-correlation method may not appropriately detect all cases of interdependencies between coupled systems. The phase-synchronization method was previously found useful in detecting synchronization in several systems. We here applied the phase-synchronization decay to study the synchronization between six combinations of binocular fixational eye movement components. We found that only two components were synchronized: the right and left horizontal with each other and the right and left vertical. Furthermore, the vertical-vertical components were much more synchronized than the horizontal.

KEYWORDS: phase synchronization decay; microsaccade; eye movements; fixation

INTRODUCTION

During fixation our eyes perform extremely small autonomic random movements, the purpose of which is assumed to be to counteract retinal adaptation. When an image is artificially fixed on the retina, it fades and disappears within a few seconds.1,2 These miniature eye movements are produced involuntarily and are characterized by three different types of movements: (1) high-frequency small-amplitude tremor, (2) slow drift, and (3) fast microsaccades.2 Drift and tremor movements are rather irregular and show statistical properties of a random walk.3 However, microsaccades create more linear movement segments embedded in the eyes’ trajectories during fixational movements.

In this study we analyzed the synchronization dynamic of binocular fixational eye movements using the phase-synchronization decay method.4-6 In such nonstationary, complex, noisy systems the traditional cross-correlation technique may not assure appropriate detection in all cases of interdependency. Our method detects weak phase synchronization between records that seem to be uncorrelated, and thus are

Address for correspondence: Ari Z. Zivotofsky, Ph.D., Gonda Brain Science Program, Bar Ilan University, Ramat Gan 52900, Israel. Voice: 972-3-531-7796; fax: 972-3-535-2184. zivotoa@mail.biu.ac.il

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likely to be from coupled systems. The motivation was to find which components (horizontal and vertical; left and right eye) of miniature eye movements are synchronized, thus implying a coupling relation.

METHODS

It is known that synchronization of weakly coupled complex systems (or oscillators) is contained in the phase and frequency relationship of the measured signals.\textsuperscript{4} The phase-difference dynamic of coupled periodic oscillators $a$ and $b$ can be described by the following equation:

$$\frac{d\varphi_{n,m}}{dt^{2}} = n\omega_{a} - m\omega_{b} + \varepsilon G(\varphi_{n,m}),$$

where $\varphi_{n,m}(t)$ is the phase difference, $\omega_{a}$ and $\omega_{b}$ are the oscillators’ frequencies, $m$ and $n$ are integers that represent the relationship between the frequencies, and $\varepsilon G(\varphi_{n,m})$ is the coupling periodic term of the oscillators. Phase synchronization deals with the phase-lock condition that has to be fulfilled:\textsuperscript{4} $|\varphi_{n,m}(t) - \delta| < \text{const}$, where $\delta$ is some average phase shift.

When the signals have relatively small noise, the phase difference $\varphi_{n,m}(t)$ changes around some constant level and the phase-lock condition is still valid. For cases of relatively large noise, there can be phase slips where the relative phase $\varphi_{n,m}(t)$ changes rapidly by $\pm 2\pi$. Thus, strictly speaking, the phase-lock condition may not be valid. However, the distribution of the cyclic relative phase, $\Psi_{n,m} = \varphi_{n,m} \mod 2\pi$, can have a dominant peak around a value corresponding to a stable fixed point. Presence of this peak can be understood as phase locking in the statistical sense.\textsuperscript{4} The sharpness of the peak of the cyclic relative phase distribution can show the amount of synchronization. This sharpness is quantified by a $\rho$ index ($0 < \rho < 1$; $0$ is not synchronized, $1$ is complete synchronization), calculated using Shannon entropy.

Phase lock is not always equivalent to coupling.\textsuperscript{7} We therefore used a modified method, the phase-synchronization decay analysis,\textsuperscript{5,6} which is based on the phase-synchronization method.\textsuperscript{4} This method detects synchronization decay and is sensitive even in cases where regular synchronization methods do not succeed. In this method we calculate the synchronization index $\rho$ as a function of shifted time ($\tau$) by shifting the signals to the “future” and “past” and computing $\rho$ in every step. The idea is that the signals had a mutual coupling “event” that was strongly concentrated in time, and by shifting we go far from this event.

A “synchronization state” is a local state. It occurs when the synchronization index versus time shift $\tau$ has two significantly falling tails and one or few maxima (see Fig. 1). A well-pronounced synchronization occurs when the difference between the tails and the maxima is more than the standard deviation of the tails. We quantify the significance by evaluation of

$$\frac{\langle \rho_{k \in \text{middle}} \rangle - \langle \rho_{k \in \text{tails}} \rangle}{\sqrt{\langle \rho_{k \in \text{tails}}^{2} \rangle - \langle \rho_{k \in \text{tails}} \rangle^{2}}}$$
Data were collected from five normal subjects, from whom permission was received, who were required to fixate a small visual stimulus. Eye movements were recorded using an EyeLink-II system (see Ref. 8). Each participant performed about 100 trials, each with a duration of 3 s.

RESULTS AND DISCUSSION

We present a new technique to study the synchronization between various components of eye movements. This technique is more sensitive for detecting weak phase synchronization. We analyzed the synchronization in fixational eye movements for all six possibilities of synchronization between eyes components. Figure 1 is the synchronization decay result for one subject for right eye horizontal and left eye horizontal, and for right eye vertical and left eye vertical, and shows a significant decay of the synchronization index. For all subjects it was found that these are the only two combinations that are synchronized. Furthermore, as seen in Figure 1, there is a significant difference in synchronization between the horizontal and vertical components, the vertical components being more synchronized than the horizontal. Figure 2 shows that these two combinations were synchronized in all five subjects and that four of the five always showed greater synchronization in the vertical that in the horizontal.

**FIGURE 1.** Synchronization decay graphs, \( \rho \) versus \( \tau \) [second]. *Upper curve* is the synchronization decay between vertical components of the two eyes; *lower curve* is between the horizontal components of the two eyes. It is seen that both components are synchronized, but the vertical shows more synchronization than horizontal.
FIGURE 2. Synchronization decay significance (considered synchronized if greater than 2) of five normal subjects. Each participant’s records were divided in half and are represented by two bars. The circles (○) present significance between vertical–vertical, and the squares (□) present significance between horizontal–horizontal.

FIGURE 3. Typical 3-s record of eye position as a function of time in the (a) vertical and (b) horizontal planes. Note that the vertical components are always conjugate (i.e., are coupled), whereas in the horizontal plane, there are regions of disconjugacy, that is, in “antiphase” (see enclosed areas).
These results suggest that the vertical movements are more coupled than the horizontal movements. FIGURE 3a shows that the vertical eye movements move together, whereas the horizontal components can at times move in “antiphase” (Fig. 3b). This difference may be based on the need for disconjugate eye movements in the horizontal plane in order to match the stereoscopic image for different viewing distance, a need never found in the vertical plane. However, we found one subject (subject 5, shown in FIGURE 2 as bars 9–10) who at times had horizontal components that were more synchronized than the vertical (bar 10), implying that this relationship is not trivial and is in need of further study.

REFERENCES