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Distribution of backbone mass between non-parallel lines

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Abstract

We determine the backbone mass distributions for bond percolation between two lines of arbitrary orientations in three dimensions. All simulations were performed at the percolation threshold p_c. The slope of the power law regime of the backbone mass distribution is dependent upon the angle between the lines, θ, but the characteristic backbone mass is only weakly affected by θ. We propose new scaling functions that reproduce the θ dependence of the characteristic backbone mass found in the simulations.

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1. Introduction

Since the 1950s, the percolation model has been applied to many disordered systems [1–4], and continues to be useful today. Here we use percolation theory to investigate the backbone mass distributions of clusters that are connected in configurations of the type shown in Fig. 1, configurations in which the two lines are connected by occupied bonds. The *backbone mass* of a cluster is the set of bonds that are connected to the

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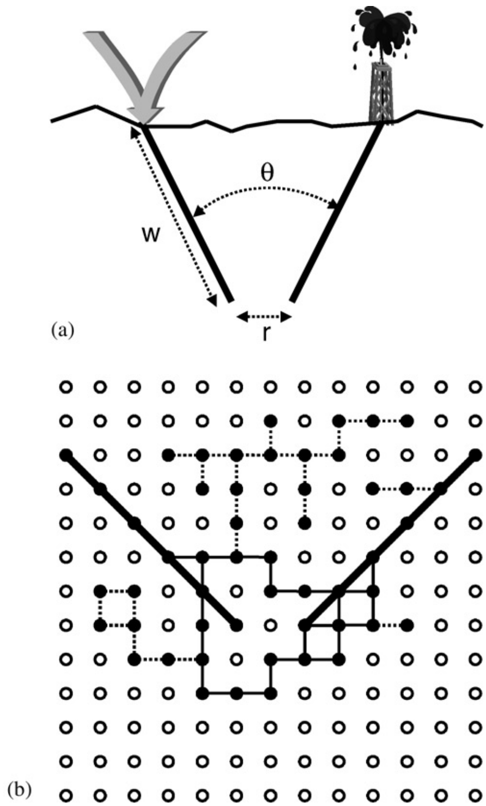

Fig. 1. (a) Illustration of well geometry. (b) Examples of a percolation cluster with two line wells with parameters $r = 2$, $\theta = 90^\circ$, and $w = \sqrt{50}$. The filled sites are members of the percolation cluster, which has a mass of 52. Solid lines form the backbone, which has a mass of 28.

two lines through independent paths (i.e., paths that have no common bond [5–8]). For configurations of two points, the distributions of various quantities have been studied [9–15]. Recently, the distribution of the shortest paths between two lines has been studied for a three-dimensional cubic lattice [13], and here we calculate the backbone distributions.

The motivation for this study is its relevance to techniques of oil recovery in oil fields [16]. A common technique used in oil recovery is the injection of fluid into the ground at one site in the field in order to force oil out of the ground at another site nearby (Fig. 1). It is common to inject the fluid along a portion of the length of the injection well and to collect the oil along a portion of the length of the production well (as opposed to injecting and collecting at single points on the wells). In our model, each line represents a well in the oil field. One line represents the injection well, and the other the production well. In many cases the oil reservoir is extremely heterogeneous, and the percolation model is appropriate. Separation of the rocks into two types—high permeability ("good rock") and low or zero permeability ("bad rock")—can be accomplished at the outset, with the good rock represented by occupied bonds and the bad rock represented by unoccupied bonds. The backbone mass represents the recoverable oil in a reservoir.

2. Simulations

We perform a numerical study of the system using Monte Carlo simulations. We specify two sets of points representing lines in a simple cubic lattice to be the wells and we grow the cluster from these two lines of seeds. If the growth of either cluster stops before the two clusters connect, we discard the realization. For realizations in which the two clusters connect, the simulation ends either when the cluster growth stops naturally, or when the cluster mass reaches some specified limit. To eliminate finite size effects, we use the techniques of Ref. [13] to simulate systems on lattices of large enough size that the clusters never reach the edge of the lattice. We perform the simulations at the percolation threshold, $p_c = 0.2488126$ [17]. The configurations are characterized by three parameters: length w, angle θ, and minimal distance r [see Fig. 1(a)]. For each configuration, we run at least 10^6 non-discarded realizations. We calculate the backbone mass for each of these realizations as exemplified in Fig. 1(b).

3. Results

In backbone mass distributions we observe an initial cutoff due to the fact that these masses cannot be smaller than the distance r. We expect to see a second cutoff due to the fact that the backbone mass cannot be greater than the cluster mass at which we stop the simulations. In addition, we expect to observe a regime that exhibits power-law behavior. These general features of the distributions have been observed in the distributions for other quantities [9–13]. The quantities of interest are (i) the most-probable value of the distribution (the maximum), the scaling of which will be determined by the fractal dimensions of the quantities measured, and (ii) the slope of the power-law regime. For clusters grown from a single point, the slopes of the power-law regimes of the backbone distributions are $\tau_B - 1$, where τ_B is the Fisher exponent for the backbone. The fractal dimensions and power law regime slopes are related by [1,3,5]

$$\tau_B - 1 = \frac{d}{d_B} , \tag{1}$$

where d is the dimension of the system, and d_B is the fractal dimensions of the backbone. For $d = 3$, estimates for these exponents are [18,19]

$$d_B = 1.855 \pm 0.015 , \tag{2}$$

$$\tau_B - 1 = 1.617 \pm 0.013 . \tag{3}$$

We show the results for the backbone probability distribution $P(m_B|\theta)$ in Fig. 2(a) for various values of θ. The distributions of backbone mass exhibit power-law regimes,

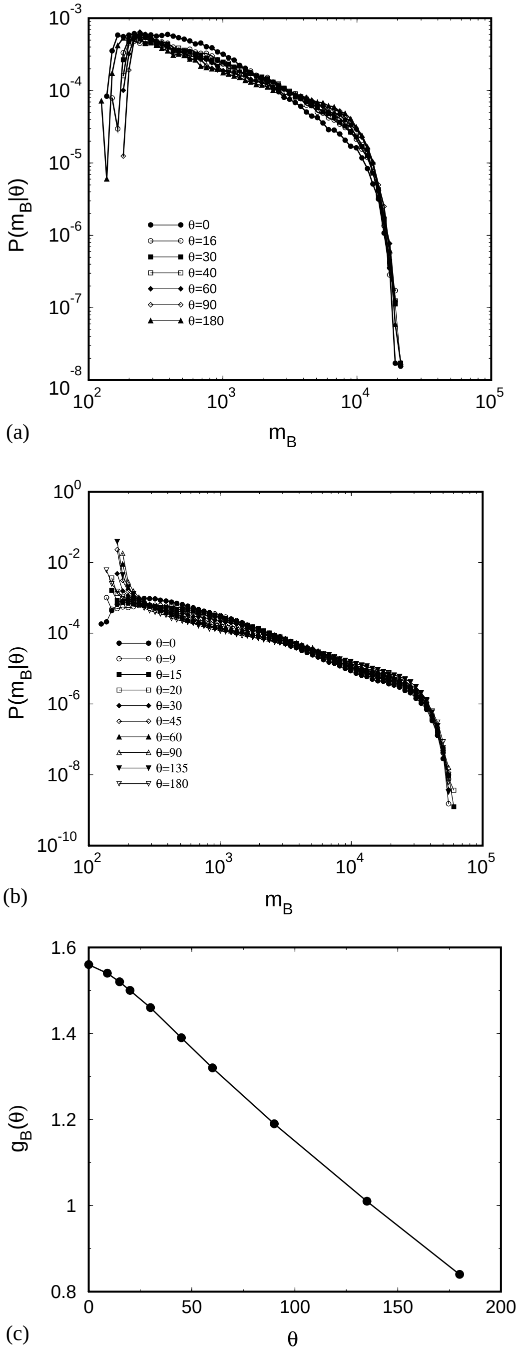

Fig. 2. (a) Backbone mass distribution $P(m_B|\theta)$ for non-parallel wells with $r=8$, $w=64$, and several values of θ. The cutoff of cluster growth is at a cluster mass of 2^{18}. (b) Backbone mass distribution $P(m_B|\theta)$ for the case of nonparallel wells with $r=1$, $w=64$ for various θ. The cutoff of cluster growth is at a cluster mass of 2^{20}. (c) Power-law exponent $g_B(\theta)$, defined in Eq. (4), for the corresponding backbone mass distribution presented in (b).

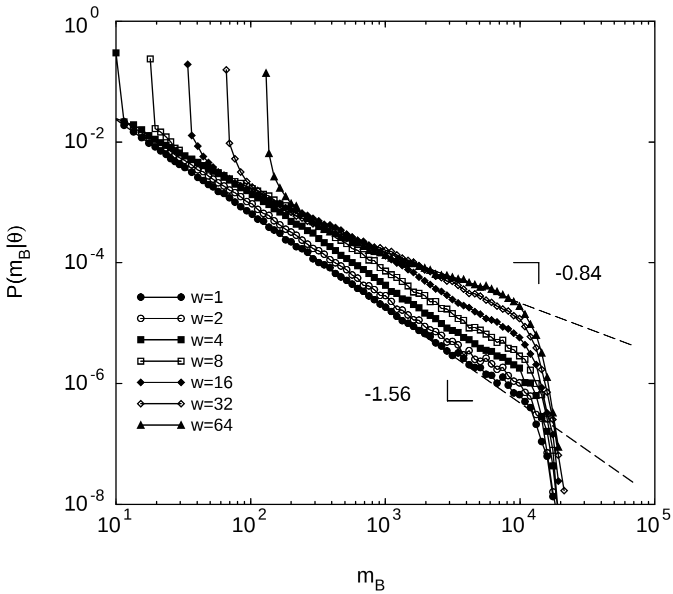

Fig. 3. Backbone mass distribution $P(m_B|w)$ for the nonparallel wells for fixed θ ($=180^\circ$) and several values of w. The larger the value of w, the later a crossover occurs from behavior reflecting a configuration of two lines with $\theta = 180^\circ$ to a configuration effectively of two points, with power-law-regime exponent $\tau_B - 1$.

the exponents of which depend on θ while the characteristic mass m^*, is essentially independent of θ. Thus, an appropriate functional form for $P(m_B|\theta)$ is

$$P(m_B|\theta) \sim \left(\frac{m_B}{r^{d_B}}\right)^{g_B(\theta)} f_1\left(\frac{m_B}{r^{d_B}}\right) f_2\left(\frac{m_B}{L^{d_B}}\right) , \tag{4}$$

where f_1 and f_2 are cutoff functions. The first cutoff function, f_1, reflects the fact that the backbone mass must always be at least equal to the distance r between the two points; the second cutoff function, f_2, reflects the fact that the backbone mass is bounded because of the finite size, L, of the system. Similar behavior has been observed for the distributions of shortest paths between two lines [13].

In order to determine the varying slope more accurately we perform simulations for various θ for $r = 1$, which results in the largest power-law regime. The results of these simulations are shown in Fig. 2(b). In Fig. 2(c) we plot the power-law regime exponent, g_B vs. θ. For $\theta = 0$, the configuration is that of parallel lines, and the slope is $\tau_B - 1$. For $\theta = \pi$, the exponent decreases to a value of about 0.84. The marked difference in these two exponents is shown clearly in Fig. 3, in which we plot for fixed $r = 1$, $\theta = 180^\circ$, $P(m_B|r)$ for various values of w. The larger the value of w, the later a crossover occurs from behavior reflecting a configuration of two lines with $\theta = 180^\circ$ to a configuration effectively of two points, with power-law-regime exponent $\tau_B - 1$.

4. Discussion

We have analyzed the distribution of backbone mass for various configurations of two-line 3d percolation clusters. We have found that the exponent of the power-law regime for the backbone mass distributions is dependent on the angle θ between the lines. It remains to develop a theory which can predict the specific dependence of this exponent on θ.

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