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Lévy flights search patterns of biological organisms

G.M. Viswanathan^{a,*}, V. Afanasyev^b, Sergey V. Buldyrev^c, Shlomo Havlin^{c,d}, M.G.E. da Luz^e, E.P. Raposo^f, H. Eugene Stanley^c

^aDepartamento de Física, Universidade Federal de Alagoas 57072-970, Maceió AL, Brazil

^bBritish Antarctic Survey, Natural Environment Research Council High Cross, Madingley Road, CB3 0ET, Cambridge, UK

^cCenter for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA

^dGonda-Goldschmied Center and Department of Physics Bar Ilan University, Ramat Gan, Israel

^eDepartamento de Física, Universidade Federal do Paraná, 81531-990, Curitiba PR, Brazil

^fLaboratório de Física Teórica e Computacional, Departamento de Física, Universidade Federal de Pernambuco, 50670-901, Recife PE, Brazil

Abstract

We discuss recent findings suggesting that an inverse square probability density distribution $P(\ell) \sim \ell^{-2}$ of step lengths ℓ leads to an optimal random search strategy for organisms that can search efficiently for randomly located objects that can only be detected in the limited vicinity of the searcher and can be revisited any number of times. We explore the extent to which these findings may be dependent on the dimensionality of the search space and the presence of short-range correlations in the step lengths and directions. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Many organisms, such as predators, must search for randomly located “target” objects (e.g., prey) whose exact locations are not known *a priori*. The question of which statistical strategy to adopt in order to optimize the random search processes has recently been studied [1]. Analytical and simulation results [1], as well as experimental data [2–5], suggest that a Lévy probability density distribution

$$P(\ell_j) \sim \ell_j^{-\mu}, \quad (1)$$

* Corresponding author. Fax: +55-82-214-1645.

E-mail address: gandhi@fis.ufal.br (G.M. Viswanathan).

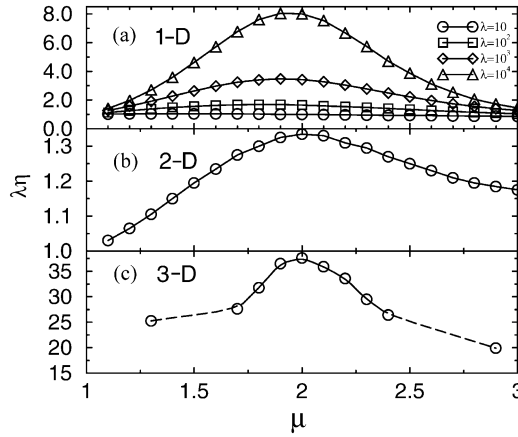


Fig. 1. The product of the search efficiency η and the mean free path λ vs. μ in for various λ , found from 1-D simulations. (b) $\eta\lambda$ found from simulations in 2-D, with $\lambda = 5000$, for a L^2 system with linear size $L = 10^4$. (c) $\eta\lambda$ for a L^3 system with $L = 100$ and a “theoretical” value $\lambda = 10\,000$. (Since $\lambda > L$, the periodic boundary conditions lead to a greater than expected value of η , because the effective value of λ is smaller than the theoretical value.) In each case, $\mu_{\text{opt}} \approx 2$ emerges as an optimal value of the Lévy flight exponent. (We have assumed $r_v = 1$ throughout.)

of step lengths ℓ_j can lead to optimal searches [1], when $\mu = 2$. Here we further discuss the dependence of this finding on the dimensionality of the search space, as well as on the presence of short-range correlations in the lengths and directions of the steps.

An analytical approach to these questions can be found in Ref. [1]. We briefly outline the argument here. Consider the following model of random search strategies: Assume that target sites are distributed randomly, and the searcher behaves as follows (see Fig. 1 in the article by da Luz et al., in this issue): (i) If there is a target site located within a “direct vision” distance r_v , then the searcher detects it with certain probability and moves on a straight line to the detected target site. (ii) If there is no detected target site within a distance r_v , then the searcher chooses a direction at random and a distance ℓ_j from the probability distribution, Eq. (1). It then incrementally moves to the new point, constantly looking for a target within a radius r_v along its way. If it does not detect a target, it stops after traversing the distance ℓ_j and chooses a new direction and a new distance ℓ_{j+1} , otherwise it proceeds to the target as in step (i) above.

This model admittedly [1] ignores predator-prey relationships and learning. Nevertheless, conclusions can still be drawn about the importance of such short-range correlated “memory” effects (see below). The analytical solution proceeds as follows: A “mean field” approximation is first obtained for the the mean free path λ of the searcher between successive target sites:

$$\langle \ell \rangle \approx \frac{\int_{r_v}^{\lambda} dx x^{1-\mu} + \lambda \int_{\lambda}^{\infty} x^{-\mu} dx}{\int_{r_v}^{\infty} x^{-\mu} dx} = \left(\frac{\mu - 1}{2 - \mu} \right) \left(\frac{\lambda^{2-\mu} - r_v^{2-\mu}}{r_v^{1-\mu}} \right) + \frac{\lambda^{2-\mu}}{r_v^{1-\mu}}.$$

Next, one defines the search efficiency function $\eta(\mu)$ to be the ratio of the number of target sites visited to the total distance traversed by the searcher, such that $\eta = [N\langle\ell\rangle]^{-1}$. N is the mean number of flights taken by a Lévy searcher in order to travel between two successive target sites. Here, we consider only the case of *nondestructive* foraging for sparsely distributed target sites, in which case a target site can be revisited any number of times. (*Destructive* foraging is treated in [1,6].) It has been shown [6,7] that in this case one obtains $N \sim (\lambda/r_v)^{(\mu-1)/2}$, for $1 < \mu \leq 3$. Substituting the expression for N and $\langle\ell\rangle$ into the definition of η and differentiating with respect to μ , one finds that the optimal efficiency $\eta = 1/(N_n\langle\ell\rangle)$ is achieved at

$$\mu_{\text{opt}} = 2 - \delta, \quad (2)$$

where $\delta \sim 1/[\ln(\lambda/r_v)]^2$. So in the absence of *a priori* knowledge about the distribution of target sites, an optimal strategy for a searcher is to choose $\mu_{\text{opt}} = 2$ when λ/r_v is large but not exactly known.

We now comment on the effects of short-range correlations in the directions and lengths of the steps. Note that the expression $N \sim (\lambda/r_v)^{(\mu-1)/2}$ describes the correct scaling even in the presence of short-range correlations. Short range correlations in the lengths of the flights are plausible because the searcher can remember how far it has traveled. Short range correlations in the directions of the flights are also plausible, because it is unlikely that an organism, for example, will suddenly make a 180° turn. Such short-range correlations can alter the width of the distribution $P(\ell)$, but cannot change μ , so the main results remain unchanged. Hence, learning, predator-prey relationships, and other short-term memory effects become unimportant in the long-time long-distance limit. We have simulated nondestructive foraging with short range correlations in the flight lengths, with correlation times up to $\tau = 10$ flights, such that step $\ell_j \equiv \ell_{j-1} \exp(-1/\tau) + \ell'_j [1 - \exp(-1/\tau)]$, where ℓ'_j is chosen according to Eq. (1). We find that the estimated value of $\mu_{\text{opt}} = 2$ remains unchanged.

Moreover, it can be argued that it would be pointless for biological organisms with even limited memory to execute Lévy flight motion with $\mu = 2$ if the optimum value $\mu_{\text{opt}} = 2$ were sensitive to the presence of such short-range correlations. The finding that microorganisms, insects, birds, and mammals have been observed to follow a Lévy distribution of flight lengths or times [1–5] thus lends support to the hypothesis that the optimum value $\mu_{\text{opt}} = 2$ is “robust” with respect to short-range “memory” effects. Indeed, the exponent μ appears to be the same in many instances [1]. When the nectar concentration is low, the flight length distribution of bumble bees [1] decays like Eq. (1) with $\mu \approx 2$. Similarly, the value $\mu \approx 2$ is also found for the foraging time distribution of the Wandering Albatross [5] and deer, in both wild and fenced areas [1].

Finally, note that the analytical results outlined above are independent of the dimension of the foraging space. Specifically, the value $\mu_{\text{opt}} = 2$ does not depend on the number of spatial dimensions. This finding is analogous to the behavior of random walks whose mean square displacement is proportional to the number of steps in any dimension [8]. We have simulated nondestructive foraging and found results (Fig. 1)

consistent with the theoretical predictions. It is interesting to note that the experimental value $2 \leq \mu \leq 2.5$ found for amoebas [3,6] is consistent with the hypothesis that $\mu_{\text{opt}} = 2$ might be a universal value of the exponent in Lévy flight foraging.

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