Percolation of Networks Composed of Connectivity and Dependency Relations

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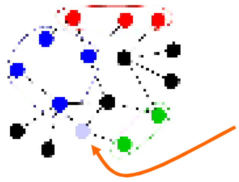
The model:

Connectivity network with dependency clusters.

The edges represent connectivity relations while the (blue, red and green) groups surrounded by curves represent dependency relations between all the nodes of the same group (color).

Cascade of failures:



Initial attack

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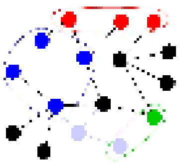

Percolation phase

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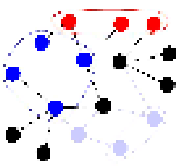

Dependency phase

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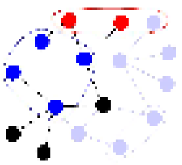

Percolation phase

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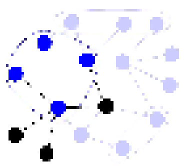

Dependency phase

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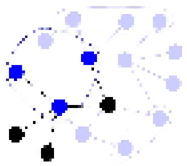

Percolation phase

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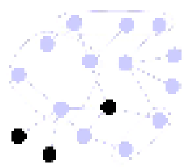


Complete fragmentation of the network.

Change from second to first order transition

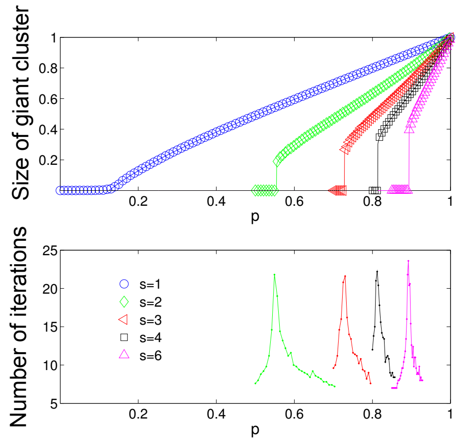
Small size of dependency clusters ⟹ Second order transition

Large size of dependency clusters ⟹ First order transition

For network with average degree, *k*, and fixed size of dependency clusters, *s*, the size of the giant cluster, P_∞ , is given by:

$$P_\infty = p^{s-1}\left(1 - e^{-kpP_\infty}\right)^s$$

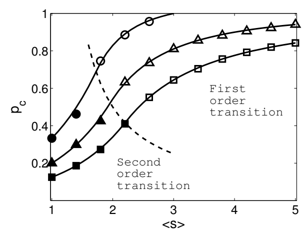
Identifying the first order transition point, p_c, by the sharp peak in the number of iterations of the cascading process.



The critical size of dependency groups

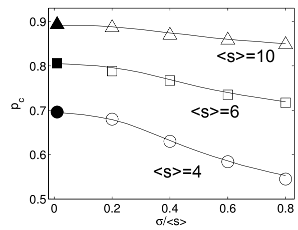
For a given network with average degree, *k*, the critical size of dependency clusters, $\langle s\rangle_c$ that leads to a change from second order to first order transition is given by:

$$2(\langle s\rangle_c - 1) \;=\; k e^{-(\langle s\rangle_c - 1)}$$



Effect of distribution width on p_c

For normally distributed dependency groups with an average size <s> and width σ the system becomes more stable for a wider distribution.



References:

S. Buldyrev, R. Parshani, G. Paul, H.E. Stanley, S. Havlin, *Catastrophic cascade of failures in coupled networks*, ***Nature***, 464, 7291 (2010).

R. Parshani, S. V. Buldyrev, S. Havlin, The critical effect of dependency groups on the function of networks, ***PNAS***, in press (2010), [arXiv:1010.4498]

A. Bashan, R. Parshani, S. Havlin, Percolation of networks composed of connectivity and dependency relations, preprint (2010).