Overlapping Synchronization

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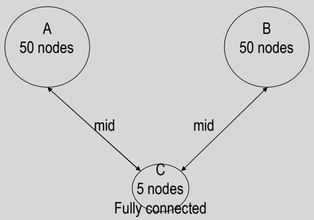
Abstract

The complex network structure has been found in many realistic systems[1]. Modularity of network (cluster),which has dense intra-connection inside and sparse interconnection between clusters, is used to explain the system function[2]. However, there usually are some overlapping structure between clusters which could be hardly partitioned to any single cluster. These overlapping clusters also play an important role in the dynamics on the complex modularity network[3]. In this paper, we mainly focus on synchronization on the overlapping structure. With Kuramoto model, we implement different natural frequency to different cluster to check how the overlapping cluster synchronizes with these clusters. It is found that with different natural frequency, the synchronization pattern of overlapping cluster could be significantly different. The result could shed some light on the understanding of how different biological functional modularity communicate.

Model

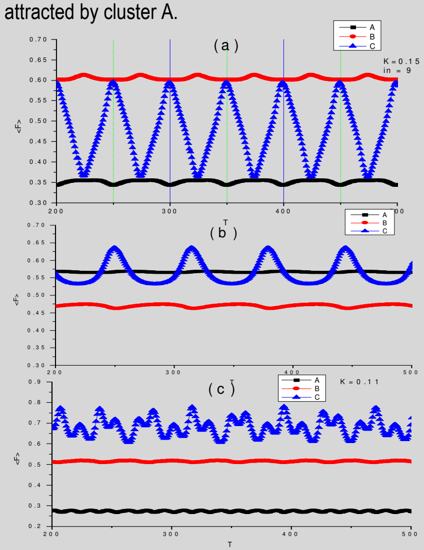
In this model, the Kuramoto model[4] is invoked to study the synchronization. The node could only be directly influenced by the other connected nodes. The network is composed of two giant cluster(ER) with different natural frequency distribution and an overlapping cluster. The connection between two giant cluster and overlapping cluster is symmetric. We wll study the synchronization frequency of each cluster.

$$\dot{\theta}_i = \omega_i + K \sum_{j \subset \Omega_i} \sin(\theta_j - \theta_i)$$



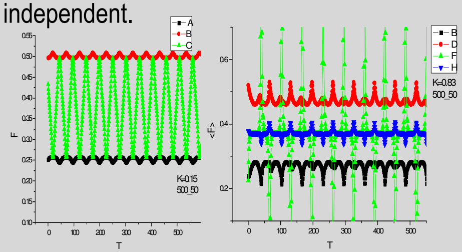
Weak Coupling

When coupling is weak, with different mean natural frequency, synchronization pattern of overlapping cluster would significantly. In Fig (a), the mean natural frequency equals to the mean frequency of the other two cluster. The real frequency oscillates symmetrically between the two clusters(Overlapping synchronization). In Fig (b), the frequency is closer to that of cluster A, while in Fig (c) the frequency is much more closer . The real frequency is more attracted by cluster A.



Large Scale

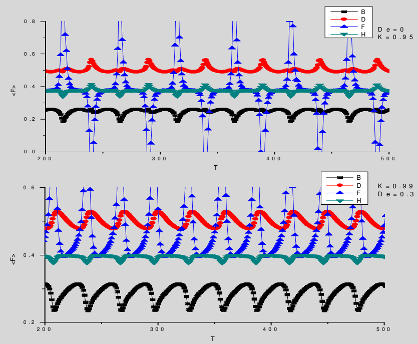
In a larger size, we still could find overlapping synchronization. It means that this kind of synchronization is size independent.



Strong Coupling

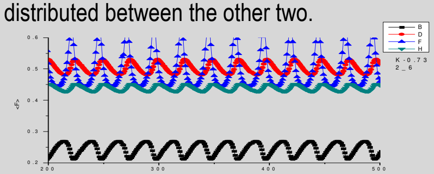
The Overlapping cluster could be regarded as coupled by mean frequency of the other two cluster. If the coupling is strong enough, the effect would be more significant. So long as the cluster(node) is overlapping structure, it would be frequency locked by mean field frequency. The closer natural frequency is from the mean field frequency, the more locking time is.

De = |Wo-(W1+ W2)/2|



Asymmetrical Coupling

If the connections from two cluster to overlapping cluster are asymmetrical. The locking frequency is also asymmetrically distributed between the other two.



Conclusion

We studied the overlapping synchronization on the overlapping network structure. It is shown that in weak coupling the mean frequency influence the synchronization pattern of overlapping cluster, while in strong coupling the influence is minimized and the frequency of overlapping cluster is always locked at the mean frequency of the two giant cluster.

Theory

$$\dot{\phi}_1 = \omega_1$$

$$\dot{\phi}_2 = \omega_2$$

The model could be approximately considered as an self-sustained oscillator coupled by two pace makers.

$$\dot{\phi}_3 = \omega_3 + K(\sin(\phi_1 - \phi_3) + \sin(\phi_2 - \phi_3))$$
$$= \omega_3 + K(\sin(\omega_1 t - \phi_3) + \sin(\omega_2 t - \phi_3))$$
$$= \omega_3 + 2K \sin(\frac{\omega_1 + \omega_2}{2} t - \phi_3) \cos(\frac{\omega_1 - \omega_2}{2} t)$$

$$\bar{\omega} = \frac{\omega_1 + \omega_2}{2}, \omega_\Delta = \frac{\omega_1 - \omega_2}{2}$$

$$\omega_3 = \bar{\omega}, \theta_3 = \phi_3 - \bar{\omega} t$$

$$\dot{\theta}_3 = 2K \cos(\omega_\Delta t) \sin \theta_3$$

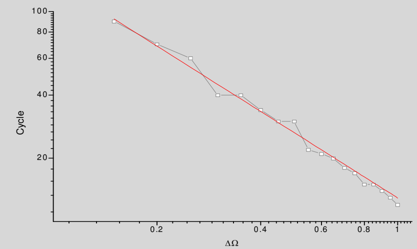
We can conclude as follows:

$$\dot{\theta}_3 = \frac{A e^{\frac{2K}{\omega_\Delta} \sin(\omega_\Delta t)} \cos(\omega_\Delta t)}{1 + B e^{\frac{4K}{\omega_\Delta} \sin(\omega_\Delta t)}}$$

In the figure below, cycle(y-axix) is for the frequency of overlapping cluster. This figure is plotted by log-log. The simulation is fitted well by theory, with the slope -1.

OverLapping frequency is ω_Δ

Note, when K increases, $\dot{\theta}_3$ would tend zero, i.e. $\dot{\phi}_3 = \bar{\omega}$



Reference

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