Traffic Flow Optimization, and the Missing Fundamental Diagram

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Evolution of the '*Fundamental Diagram*'

Traffic macroscopic properties

ρ - density of cars
f - total flux on the road
v - average cars speed

Relations:

(1) $f = v \cdot \rho$
(2) $f = f(\rho)$ ⇨ '*Fundamental Diagram'*

Open questions

For many decades, it was believed that the density-flux relation is consisted of 1-2 curves (Figs. 1a-b).

The recent empirical study of Kerner [1] shows that such **fundamental diagram does not exist**. Instead, congested traffic displays a **2D region in the density-flux plane** (Fig. 1c), **unexplained by models**. That is, for a given value of density, there exists a range of possible flux values.

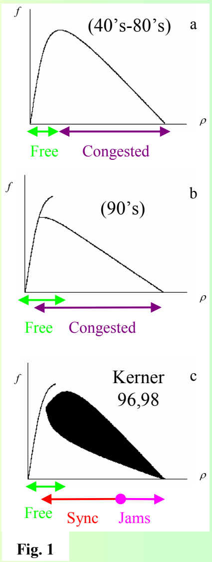


Fig. 1

Mechanisms to optimize traffic flow by approaching the highest values of this range have not yet been suggested

Traffic flow optimization in systems with a localized periodic inhomogeneity

We study whether this new insight into the nature of traffic flow can be applied to optimizing the flux close to its maximal value for a given congested density. For this purpose, systems with periodic localized inhomogeneities are studied, using a recent car-following model [2]. Two types of periodic inhomogeneities are considered, and in both cases we focus on oversaturation, where traffic is congested upstream of these inhomogeneities.

Signalized intersection

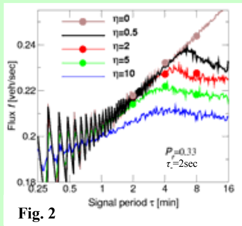
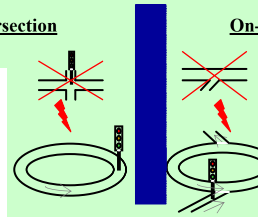


Fig. 2



τ - signal period,
P_r - relative period of red light,
η - white noise amplitude,
τ_- - the total time in each period where the intersection is empty.

Unexpected phenomena:

Traffic flow theory: $f(\tau)=f_0(1-\tau_-/\tau)\sim const.$
Deterministic ($\eta=0$) model: *f grows with τ*
Stochastic ($\eta>0$) model: **an optimal τ exist !**

For ($\eta>0$), a **crossover** is observed from the deterministic $f(\tau)$ to lower values of flux.

The optimal value of τ approaches the crossover point for small η.

On-ramp with signalized entrance

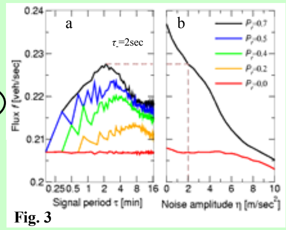


Fig. 3

Note that unlike the signalized intersection, where P_r is predetermined, here both P_r and τ are optimization parameters (Fig. 3a).

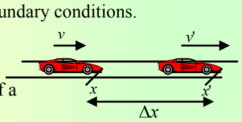
The increase in flux due to the introduction of traffic lights varies from 1.0% for η=10 through **10.0%** for η=2 to **13.9%** for η=0 (Fig. 3b)..

This significant improvement is obtained **without causing congestion on the secondary road !**

Model assumptions

To explain both our results on flux optimization and the experimental density-flux relation, we study our inertial car-following model [2] in a single lane homogenous system with periodic boundary conditions.

Inertial car-following models usually assume that the acceleration a of a car is $a = F(\Delta x, \Delta v, v)$



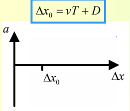
In this model we assume that the interaction between consecutive cars is affected by the following factors:

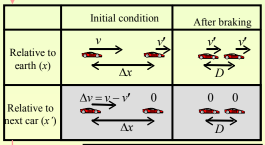
1. Desire to keep a safety time gap T
2. Anticipation: pre-braking if the car ahead is slower
3. Coupling to the permitted velocity v_{per} (speed limit)
4. Randomization

$$a = A\left(1-\frac{\Delta x_0}{\Delta x}\right) - \frac{(\Delta v)^2}{2(\Delta x - D)} - k(v - v_{per}) + \eta$$

if $v<v'$ if $v>v_{per}$

$\Delta x_0 = vT + D$





$$a = \left.\frac{v^2 - v_0^2}{2(x - x_0)}\right|_{x'} = -\frac{(\Delta v)^2}{2(\Delta x - D)}$$

In [2] we showed how this model can be used to demonstrate and explain many of the empirically found phenomena of traffic, such as bistability of free and congested traffic, hysteresis in transitions between free and congested states, and more.

The generalized *Fundamental Diagram*

The system of ordinary differential equations describing the motion for the deterministic model have two types of stable states:

1. Homogeneous flow - fixed point in phase space (low / high densities),
2. Multistable periodic states - limit cycles in phase space (intermediate densities).

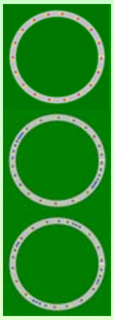


Fig. 4a show that the **fundamental diagram** has to be generalized:

$$f = f(\rho) \longrightarrow f = f(\rho, \lambda)$$

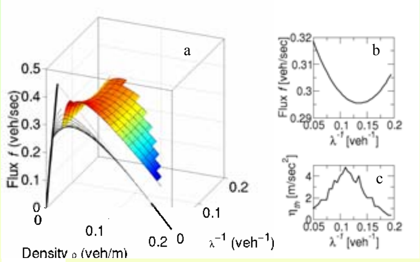


Fig. 4

⇨ **2D region in density-flux plane is a projection of Fig. 4a surface on the density-flux plane** (thin curves).

An example of wavelength-flux relation is displayed in Fig 4b. Fig 4c shows the noise stability threshold amplitude, above which the states presented in Fig. 4b become unstable. Apparently states with relatively low values of flux are the most stable in the presence of noise. Therefore real-life stop-and-go traffic is expected to show frequently values of flux far below the optimum.

The optimization process

This new insight on traffic enables to explain the previous optimization results:

For signalized intersection with η=0 (Fig. 2), increasing τ ⟶ increasing λ ⟶ higher flux (Fig. 5a) (Fig. 4b)

For η>0, the crossover in Fig. 2 is related to crossing the noise threshold η_{th} (see Fig. 4c). Optimal τ is usually close to crossover point. Above this point small jams emerge, causing change in effective λ. The nine space-time diagrams in Fig. 5a, corresponding to the nine circles in Fig. 2, visualize this transition. This explanation is supported by the fact that autocorrelation functions is $ac_v(\tau)=1$ below the crossover, and $ac_v(\tau)<1$ above it (Fig. 5b).

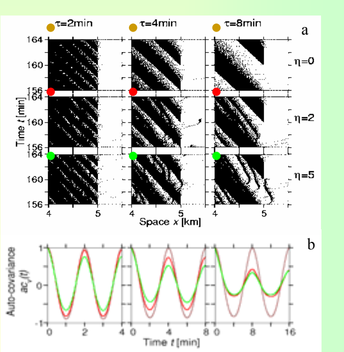


Fig. 5

References

[1] B.S. Kerner, *Phys. Rev. Lett.* **81**, 3797 (1998).
[2] E. Tomer, L. Safonov, and S. Havlin, *Phys. Rev. Lett.* **84**, 382 (2000).
[3] Tomer, Safonov, Madar, and Havlin, *Preprint.*