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978-0-521-84156-6 - Complex Networks: Structure, Robustness and Function

Reuven Cohen and Shlomo Havlin

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## Complex Networks

### Structure, Robustness and Function

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Examining important results and analytical techniques, this graduate-level textbook is a step-by-step presentation of the structure and function of complex networks.

From the stability of the Internet to efficient methods of immunizing populations, from epidemic spreading to how to efficiently search for individuals, this textbook explains the theoretical methods used, and the experimental and analytical results obtained. Ideal for graduate students and researchers entering this field, it gives detailed derivations of many results in complex networks theory. End-of-chapter review questions help students monitor their understanding of the materials presented.

**Reuven Cohen** is a Senior Lecturer in the Department of Mathematics at Bar-Ilan University, Israel. He has written many papers in the fields of complex networks, robot swarms, algorithms and communication networks, and has won several national and international prizes for his work.

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